21-128 and 15-151 problem sheet 10

Solutions to the following 8 exercises are to be submitted through blackboard by 8:30AM on

Tuesday 6th December 2016.

There are three extra problems for additional practice.

Problem 1

Let A and B be events in a probability space S. Is it true that if $A \subseteq B$ then $\mathbb{P}(A) \leq \mathbb{P}(B)$? If true, provide a proof; if false, provide a counterexample.

Problem 2

Let A and B be events in a probability space S. Is it true that if A and B are independent, then A^c and B^c are independent? If true, provide a proof; if false, provide a counterexample.

Problem 3

Each of three containers has two marbles; one contains two red marbles, one contains two black marbles, and one contains one red and one black marble. A container is selected at random (each equally likely), and one of the two marbles inside is selected at random (each equally likely). Given that the selected ball is black, what is the probability that the other ball in its container is black?

Problem 4

We roll two dice, one red and one green. Under each assumption below, what is the probability that two sixes are rolled?

(a) The red die shows a six.

(b) At least one of the dice shows a six. Does the method of obtaining this information affect the answer?

Problem 5

The proportion of the games that a tennis player wins against each of her four opponents is 60%, 50%, 45% and 40%, respectively. Suppose that she plays 30 matches against each of the first two and 20 matches against each of the last two. Given that she wins a particular match, what is the conditional probability that it is against the i^{th} opponent, for $i \in \{1, 2, 3, 4\}$?

Problem 6

A drunk has n keys, and only one will open the door. He tries keys at random. Under each model below, what is the expected number of selections until he opens the door?

(a) He selects keys in a random order (without replacement) until one works.

(b) After each mistake, he replaces the key and selects randomly again.

Problem 7

Suppose that n pairs of socks are put into the laundry, with each sock having one mate. The laundry machine randomly eats socks; a random set of k socks returns. Determine the expected number of complete pairs of returned socks.

(Hint: use linearity of expectation.)

Problem 8

Suppose that $2n$ people are partitioned into pairs at random, with each partition being equally likely. If the set consists of n men and n women, what is the expected number of male-female couples?

Extra Problem 1

Half of the women and one third of the men in a class are smokers. Two thirds of the students are men. What proportion of the smokers are women?

Extra Problem 2

Suppose that Person A, Person B, and n other people stand in a line in random order. For each k with $0 \leq k \leq n$, find the probability that exactly k people stand between Person A and Person B. Verify, with proof, that the sum of these probabilities equals 1.

Extra Problem 3

Read Application 9.31 on page 181 of the book, and then answer the following questions.

(a) Which values of $x \in [0, 1]$ guarantee a positive expectation for Player B no matter what Player A does?

(b) We have seen that when each player shows one finger with probability $\frac{7}{12}$, Player B expects to win with an average of $\frac{1}{12}$ dollars per game. With these strategies, what *proportion* of the games does Player B expect to win?