21-256 Homework 2

Due Friday 23rd May 2014

1. Find the equation of a sphere if one of its diameters has end-points $(2,1,4)$ and $(4,3,10)$.

2. Describe in words the region of $\mathbb{R}^3$ represented by the equation $x = z$.

3. Find $\mathbf{v}+\mathbf{w}$, $2\mathbf{v}+3\mathbf{w}$, $\|\mathbf{v}\|$ and $\|\mathbf{v}-\mathbf{w}\|$ when $\mathbf{v} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$.

4. Find $\mathbf{a}+\mathbf{b}$, $2\mathbf{a}+3\mathbf{b}$, $\|\mathbf{a}\|$ and $\|\mathbf{a}-\mathbf{b}\|$ when $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$.

5. Find $(2\mathbf{i}+\mathbf{j}) \cdot (\mathbf{i}-\mathbf{j}+\mathbf{k})$.

6. Find the acute angle between the vectors $\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$.

7. Find the scalar and vector projections of $\begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}$ onto $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

8. Find the scalar and vector projections of $\mathbf{i}+\mathbf{j}+\mathbf{k}$ onto $\mathbf{i}-\mathbf{j}+\mathbf{k}$.

9. Compute $(\mathbf{j}+7\mathbf{k}) \times (2\mathbf{i}-\mathbf{j}+4\mathbf{k})$ and verify that it is orthogonal to both $\mathbf{j}+7\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$.

10. Find the volume of the parallelepiped with adjacent edges PQ, PR and PS, where the points P, Q, R, S are given by

$$P = (-2,1,0), \quad Q = (2,3,2), \quad R = (1,4,-1), \quad S = (3,6,1)$$

Extra credit problems

E1. Prove that $\|\mathbf{a}+\mathbf{b}\| \leqslant \|\mathbf{a}\| + \|\mathbf{b}\|$ for all n-dimensional vectors $\mathbf{a}, \mathbf{b}$, and give a geometric interpretation. Under what conditions is it true that $\|\mathbf{a}+\mathbf{b}\| = \|\mathbf{a}\| + \|\mathbf{b}\|$?

E2. Prove that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ for all 3-dimensional vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and use this result to prove that

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$$

Questions are worth 10 points each. Extra credit problems are optional and will only be counted for credit if you submit answers to all other questions; they can be used to raise your total score to a maximum of 100 points.