Math 290-1 Class 11 — review for midterm 1

Monday 22nd October 2018

1. Find the matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects each vector through the line $2x + 3y = 0$.

[*You may use the fact that the reflection of a vector* $\vec{x}$ *through a line* ℓ *with direction* $\vec{a}$ *is given by the formula* $\operatorname{ref}_\ell(\vec{x}) = 2\left(\dfrac{\vec{x}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a} - \vec{x}$.]

Is T invertible? If not, explain why; if so, find the matrix of T^{-1}.

2. For each of the following statements about $n \times n$ matrices A, B and C, determine whether it is always true, sometimes true, or never true.

(a) If $AB = C$, then $B = CA^{-1}$.

(b) For each vector $\vec{v}$ in $\mathbb{R}^n$, the vector $A\vec{v}$ is a linear combination of the columns of A.

(c) $\text{rank}(AB) = \text{rank}(A)\text{rank}(B)$

(d) If $\text{rank}(A) < n$, then the system $A\vec{x} = \vec{0}$ has infinitely many solutions.

(e) If ABC is invertible, then B is invertible.

(f) If A is the matrix of orthogonal projection onto a line, then $A^2 \neq A$.

3. (a) Find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 2 & 3 & -5 \\ -1 & -1 & 2 \end{pmatrix}$

(b) Express $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ as a linear combination of the vectors $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -5 \\ 2 \end{pmatrix}$

4. Find the matrices of the linear transformations $T \circ S$ and $S \circ T$, where $S : \mathbb{R}^3 \to \mathbb{R}^2$ projects each vector onto its first two coordinates and $T : \mathbb{R}^2 \to \mathbb{R}^3$ embeds $\mathbb{R}^2$ into the (x,z)-plane:

$$S \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ 0 \\ b \end{pmatrix}$$