

We will focus on the exercises from Wednesday. If there's time, we might get to this one—if not, see Canvas for the worked solution!

1. Find a basis for the subspace of  $\mathbb{R}^5$  consisting of all linear combinations of the following vectors:

$$\begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \\ -7 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \\ -1 \\ -4 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -2 \\ 4 \\ 3 \end{pmatrix}$$

We put the matrix whose columns are these vectors into ref & keep the vectors corresponding to pivots.

$$\begin{pmatrix} \textcircled{1} & 1 & 7 & -2 & 0 & 7 & 0 \\ 2 & 0 & -2 & 2 & 0 & 2 & 2 \\ 1 & -2 & -7 & 2 & 0 & -1 & -2 \\ 0 & 1 & -2 & 2 & 0 & -4 & 4 \\ -1 & 1 & -1 & 1 & 0 & -5 & 3 \end{pmatrix}$$

$$\begin{array}{l} \text{(II)} - 2\text{(I)} \\ \text{(III)} - \text{(I)} \\ \text{(V)} + \text{(I)} \end{array} \begin{pmatrix} \textcircled{1} & 1 & 7 & -2 & 0 & 7 & 0 \\ 0 & -2 & -16 & 6 & 0 & -12 & 2 \\ 0 & -3 & -14 & 4 & 0 & -8 & -2 \\ 0 & \textcircled{1} & -2 & 2 & 0 & -4 & 4 \\ 0 & 2 & 6 & -1 & 0 & 2 & 3 \end{pmatrix}$$

$$\begin{array}{l} \text{(II)} + 2\text{(IV)} \\ \text{(III)} + 3\text{(IV)} \\ \text{(V)} - 2\text{(IV)} \end{array} \begin{pmatrix} \textcircled{1} & 1 & 7 & -2 & 0 & 7 & 0 \\ 0 & 0 & -20 & 10 & 0 & -20 & 10 \\ 0 & 0 & -20 & 10 & 0 & -20 & 10 \\ 0 & \textcircled{1} & -2 & 2 & 0 & -4 & 4 \\ 0 & 0 & \textcircled{10} & -5 & 0 & 10 & -5 \end{pmatrix}$$

$$\begin{array}{l} \text{(II)} + 2\text{(V)} \\ \text{(III)} + 2\text{(V)} \end{array} \begin{pmatrix} \textcircled{1} & 1 & 7 & -2 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \textcircled{1} & -2 & 2 & 0 & -4 & 4 \\ 0 & 0 & \textcircled{10} & -5 & 0 & 10 & -5 \end{pmatrix}$$

At this point we can tell that there will be pivots in columns 1, 2 and 3 in the ref of the matrix

$$\Rightarrow \text{the basis is given by } \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \\ -7 \\ -2 \\ -1 \end{pmatrix}.$$