

Math 290-1 Class 20 — review for midterm 2

Monday 12th November 2018

1. For each of the following statements, determine whether it is always, sometimes or never true.

(a) Let A be a 4×5 matrix with rank 4. There is a 4×3 matrix B and a 3×5 matrix C such that $A = BC$.

(b) Let V be a subspace of \mathbb{R}^3 . Then $V = \ker(T)$ for some linear transformation T .

(c) Let \mathcal{A} and \mathcal{B} be distinct bases of \mathbb{R}^2 . For every linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, the \mathcal{A} -matrix of T is equal to the \mathcal{B} -matrix of T .

2. For each of the following questions, determine whether it is true or false.

(a) Let \vec{a} and \vec{b} be vectors in \mathbb{R}^n . If \vec{c} is in $\text{span}(\vec{a}, \vec{b})$, then $T(\vec{c})$ is in $\text{span}(T(\vec{a}), T(\vec{b}))$.

(b) The matrices $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$ are similar.

(c) The transition matrix of a basis \mathfrak{B} of \mathbb{R}^n is invertible.

3. Let k be some number. In terms of k , find the dimension of the image of the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$$T(\vec{x}) = \begin{pmatrix} 1 & k & k^2 \\ k & 3k^2 & 2k^3 \\ 1 & 3k & 2k^2 \end{pmatrix} \vec{x}$$

4. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation defined by rotation by an angle θ about a line ℓ through the origin, where $0 \leq \theta < 2\pi$. Give a geometric description of a basis of \mathbb{R}^3 with respect to which the matrix of T is of the following form:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{pmatrix}$$

and find the values of a , b , c and d in this case.

For which values of θ is this matrix diagonal?