1. Find the equation of the circle \((x - 3)^2 + y^2 = 9\) in polar coordinates.

\[
\begin{align*}
(x - 3)^2 + y^2 &= 9 \\
\Rightarrow x^2 - 6x + 9 + y^2 &= 9 \\
\Rightarrow x^2 + y^2 &= 6x \\
\Rightarrow r^2 &= 6r \cos \theta \\
\Rightarrow r &= 6 \cos \theta
\end{align*}
\]

Note: \(r < 0\) when \(\pi < \theta < 2\pi\), so for these values of \(\theta\), the point \((r, \theta)\) lies on the ray opposite the ray at angle \(\theta\).

2. Sketch the curve in \(\mathbb{R}^2\) described in polar coordinates by the equation \(r = \theta\) (for \(r \geq 0\)).
3. (a) Sketch the surface in $\mathbb{R}^3$ whose equation in cylindrical coordinates is $z = 2r$.

(b) Find the cartesian coordinates of the surface you just sketched.

\[ z = 2r \Rightarrow z^2 = 4r^2 \]

\[ z^2 = 4(x^2 + y^2) \]

\[ 4x^2 + 4y^2 - z^2 = 0 \]

>Note: $4x^2 + 4y^2 - z^2 = x^T A x$

Where $A = \begin{pmatrix} 4 & 4 \\ 4 & -1 \end{pmatrix}$

The surface $z = 2r$ is therefore a (degenerate) hyperboloid — more on this next week.
4. Sketch the solid region of $\mathbb{R}^3$ described in spherical coordinates by

$$0 \leq \rho \leq 1, \quad 0 \leq \varphi \leq \frac{\pi}{4}, \quad 0 \leq \theta \leq \pi$$

5. Find a way of converting between cylindrical and spherical coordinates.

Cylindrical to spherical

$$\begin{align*}
\rho^2 &= r^2 + z^2 \\
\tan \varphi &= r \quad \Rightarrow \quad \varphi = \tan^{-1} \left( \frac{r}{z} \right) \\
\theta &= \theta
\end{align*}$$

Spherical to cylindrical

$$\begin{align*}
r &= \rho \sin \varphi \\
z &= \rho \cos \varphi \\
\theta &= \theta
\end{align*}$$