Northwestern University Name:

Math 290-1: Midterm 1

Fall Quarter 2014

Monday, October 20, 2014

Put a check mark next to your section:

Davis (10am)		Canez	
Alongi		Peterson	
Graham		Davis (12pm)	

Question	Possible points	Score
1	20	
2	20	
3	10	
4	15	
5	15	
6	20	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- This exam has 11 pages, and 6 questions. Please make sure that all pages are included.
- You may not use books, notes or calculators.
- You have one hour to complete this exam.

Good luck!

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This problem has **four** parts.)

   (a) Suppose that the system

$$\begin{aligned} ax_1 + bx_2 + cx_3 &= d \\ ex_1 + fx_2 + gx_3 &= h \end{aligned}$$

is consistent. (The variables are x_1, x_2, x_3.) If the system

$$\begin{aligned} ax_1 + bx_2 + cx_3 &= d \\ ex_1 + fx_2 + gx_3 &= h \\ ix_1 + jx_2 + kx_3 &= l \end{aligned}$$

has the same set of solutions as the first system, then the equation

$$ix_1 + jx_2 + kx_3 = l$$

is a multiple of one of the two equations in the first system.

Answer:

(b) There is a scalar k which makes the function $T : \mathbb{R}^2 \to \mathbb{R}^4$ given by

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y\cos((k^2-1)x) \\ 3x + (2k-1)^2 y \\ 3 \\ (k+1)x^2 - 4y \end{bmatrix}$$

a linear transformation.

Answer:

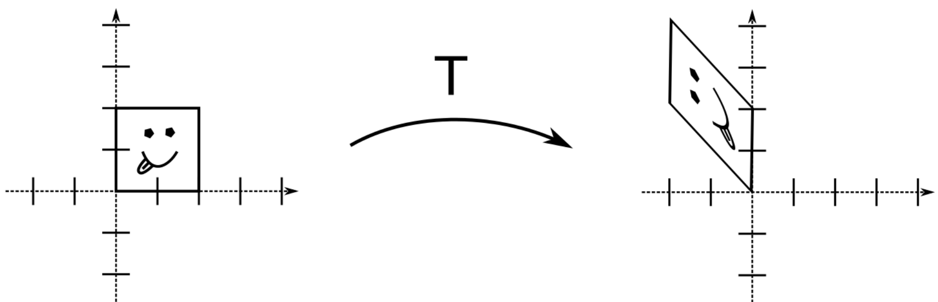
(c) If A and B are $n \times n$ invertible matrices, then AB has rank n.

Answer:

(d) Applying the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by the matrix product

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

transforms the smiley face on the left into the smiley face on the right:



Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This problem has **four** parts.)

   (a) Let k be a real number. The matrix

   $$\begin{bmatrix} 1 & 1+k^2 & 1+2k^2 \\ 0 & 1+k^2 & 1+2k^2 \\ 0 & 0 & 1+2k^2 \end{bmatrix}$$

   has rank 3.

Answer:

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. The equation $T(\vec{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions.

Answer:

(c) Suppose that $\vec{u}$ and $\vec{v}$ are nonzero perpendicular vectors in $\mathbb{R}^2$. Then any vector $\vec{b}$ in $\mathbb{R}^2$ is a linear combination of $\vec{u}$ and $\vec{v}$.

Answer:

(d) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ be an invertible matrix. Then the matrix $B = \begin{bmatrix} b & a & c \\ e & d & f \\ h & g & i \end{bmatrix}$ is also invertible.

Answer:

3. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$. Find conditions on the scalars b_1, b_2, b_3 so that $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is **not** a linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first rotates $\mathbb{R}^2$ by π radians, then applies the shear determined by $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, then reflects across the line $y = -x$, and finally scales by a factor of 3. Find the matrix of T.

5. Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$. Find a 2×2 matrix B such that B is not the zero matrix and AB is the zero matrix.

6. (This problem has **two** parts.) Let A be the following matrix.

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

(a) Find the inverse of A.

(b) Find the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ satisfying

$$T\begin{bmatrix}-1\\0\\2\end{bmatrix} = \begin{bmatrix}3\\0\\0\end{bmatrix}, \quad T\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}2\\1\\0\end{bmatrix}, \quad \text{and} \quad T\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\3\\0\end{bmatrix}.$$

Hint: Notice that the given input vectors are precisely the columns of the matrix A defined previously. Try using the inverse you found in part (a).