Northwestern University **Name:**

Math 290-3: Midterm 1

Spring Quarter 2015

Thursday, April 30, 2015

Put a check mark next to your section:

Davis (10am)		Canez	
Peterson		Davis (12pm)	

Question	Possible points	Score
1	20	
2	20	
3	15	
4	15	
5	15	
6	15	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- This exam has 9 pages, and 6 questions. Please make sure that all pages are included.
- You may not use books, notes or calculators.
- You have one hour to complete this exam.

Good luck!

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This problem has **four** parts.)

   (a) The following integrals are equal:

$$\int_0^{2\pi} \int_0^R \int_r^R \theta\, r\, dz\, dr\, d\theta \quad \text{and} \quad \int_0^{2\pi} \int_0^{\pi/4} \int_0^{R/\cos\phi} \theta\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta.$$

Answer:

   (b) Suppose that R is a rectangle in $\mathbb{R}^2$ and that $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous. If $\iint_R f(x,y)\, dA = 0$, then every Riemann sum of f has the value zero.

Answer:

(c) The following equality holds:

$$\int_0^4 \int_{-1}^1 \int_0^2 (4 - 4y + y^{101} e^{\cos(y^2 z) + \sin x + z})\, dz\, dy\, dx = \frac{8}{3}$$

Answer:

(d) The following equality holds:

$$\int_0^1 \int_0^x dy\, dx + \int_1^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} dy\, dx = \frac{\pi}{4}$$

Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This problem has **four** parts.)

   (a) For a number k,

$$\int_{10}^{20}\int_{0}^{e^x} [x^2 + y^2 + (k^2 + 1)^3]\, dy\, dx \geq 0.$$

Answer:

   (b) For a continuous function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$\int_0^1 \int_{-x}^{x} f(x, y) dy dx = \int_{-1}^{1} \int_0^1 v f(v, uv) dv du.$$

Answer:

(c) For $k > 0$, if E is the solid consisting of all points satisfying the inequalities $k \leq \sqrt{x^2 + y^2 + z^2} \leq 2k$, then

$$\iiint_E \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \, dV = 4\pi^2 \ln(2).$$

Answer:

(d) For a number k,

$$\int_0^2 \int_0^x k e^{kx^{10}} dy dx = \int_0^1 \int_y^{2-y} e^{kx^{10}} dx dy + \int_1^2 \int_{2-x}^x e^{kx^{10}} dy dx.$$

Answer:

3. Determine the value of the following expression.

$$\int_{-1}^{0}\int_{-x}^{1} e^{y^2}\,dy\,dx + \int_{0}^{1}\int_{x}^{1} e^{y^2}\,dy\,dx$$

4. Suppose that $f : \mathbb{R}^3 \to \mathbb{R}$ is a continuous function. Rewrite the following triple integral with respect to the order $dy\,dx\,dz$.

$$\int_0^1 \int_0^{1-y} \int_0^{1-y} f(x,y,z)\,dx\,dz\,dy$$

5. Find a change of variables $u = u(x, y)$ and $v = v(x, y)$ under which the double integral $\iint_D (6y + 3)\, dA$, where D is the region in the xy-plane bounded by the curves

$$y = -x,\ \ y = 1 - x,\ \ x = y^2,\ \text{ and } x = y^2 - 1,$$

becomes the double integral of a constant over a rectangle in the uv-plane. Setup the integral in terms of u and v, but do NOT evaluate it.

6. Determine the value of

$$\iiint_E \frac{1}{x^2 + y^2 + z^2}\, dV$$

where E is the region above the xy-plane which lies outside the sphere $x^2 + y^2 + z^2 = 2$ and inside the sphere $x^2 + y^2 + z^2 = 2z$.