Northwestern University **Name:**

Math 290-2: Midterm 1

Winter Quarter 2015

Monday, February 2, 2015

Put a check mark next to your section:

Davis		Canez	
Alongi		Peterson	

Question	Possible points	Score
1	20	
2	20	
3	10	
4	20	
5	10	
6	20	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- This exam has 11 pages, and 6 questions. Please make sure that all pages are included.
- You may not use books, notes or calculators.
- You have one hour to complete this exam.

Good luck!

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This problem has **four** parts.)

   (a) If A is any square matrix and $q(\mathbf{x}) = \mathbf{x} \cdot A\mathbf{x}$ is a quadratic form, then A must be diagonalizable.

Answer:

   (b) If A is a symmetric matrix and $\mathbf{v}$ and $\mathbf{w}$ and vectors with $\mathbf{v}$ in the kernel of A and $\mathbf{w}$ in the image of A, then $\mathbf{v} \cdot \mathbf{w} = 0$.

Answer:

(c) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are any nonzero vectors in $\mathbb{R}^3$, then $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{c} \times \mathbf{b}) \cdot \mathbf{a}$.

Answer:

(d) If V is a nonzero subspace of $\mathbb{R}^n$, then V has an orthonormal basis.

Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This problem has **four** parts.)

   (a) Suppose that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthonormal set of vectors in $\mathbb{R}^n$. Then

   $$\{\mathbf{u}_1 - \mathbf{u}_2, \mathbf{u}_2 - \mathbf{u}_3, \mathbf{u}_1 - \mathbf{u}_3\}$$

   is also an orthonormal set.

Answer:

   (b) For a square matrix Q whose columns are perpendicular to one another, $Q^T Q = I$.

Answer:

(c) For an $n \times m$ matrix A and a vector $\mathbf{b}$ in $\mathbb{R}^n$, there is a vector $\mathbf{x}$ in $\mathbb{R}^m$ such that $A\mathbf{x} = \operatorname{proj}_{\operatorname{im} A} \mathbf{b}$.

Answer:

(d) For a number k, the line with parametric equations

$$x = 4 - 2t,\ y = k + t,\ z = 2k - 1$$

intersects the surface described by the equation $(x-4)^2 + (y-k)^2 + (z+1)^2 = 1$.

Answer:

3. At O'Hare airport, the average low temperature (in degrees Celsius) t months into the year is listed below:

t	**°C**
0	-8
3	5
6	20
9	8

Find the function of the form $f(t) = c_0 + c_1 \sin(\frac{\pi t}{6}) + c_2 \cos(\frac{\pi t}{6})$ which best fits the data above, using least squares.

4. (This problem has **three** parts.) Define the quadratic form $q : \mathbb{R}^2 \to \mathbb{R}$ by

$$q(x, y) = 4x^2 + 4xy + 7y^2.$$

(a) Determine whether q is positive definite, negative definite, or indefinite.

(b) Find a set of principal axes for q.

(c) Draw the curve whose equation is $4x^2 + 4xy + 7y^2 = 1$, labeling the principal axes and the intercepts of the curve with these axes. The intercepts should be labeled by their standard (x, y) coordinates.

5. Let P be the plane $x + 2y + 4z = 0$ and let $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$. Find the distance from $\mathbf{x}$ to P.

6. (This problem has **two** parts.) Suppose that A is a 4×4 matrix with eigenvalues 2 and -1, and associated eigenvectors

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}-2\\0\\0\\-2\end{bmatrix}, \begin{bmatrix}1\\4\\2\\1\end{bmatrix} \text{ for 2 and } \begin{bmatrix}1\\0\\0\\-1\end{bmatrix} \text{ for } -1.$$

(a) Find a basis of $\mathbb{R}^4$ consisting of orthonormal eigenvectors of A.

(b) Compute $A^3 \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ explicitly.