Northwestern University **Name:**

Math 290-2: Midterm 2

Winter Quarter 2015

Monday, March 2, 2015

Put a check mark next to your section:

Davis		Canez	
Alongi		Peterson	

Question	Possible points	Score
1	20	
2	20	
3	15	
4	10	
5	15	
6	20	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- This exam has 10 pages, and 6 questions. Please make sure that all pages are included.
- You may not use books, notes or calculators.
- You have one hour to complete this exam.

Good luck!

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This problem has **four** parts.)

   (a) The graph of $\rho = 1 - \sin(\phi)$ describes a sphere.

Answer:

   (b) There exist numbers k and ℓ such that level sets of the functions $f(x, y, z) = x+y+z$ and $g(x, y, z) = x + y + z + 1$ at levels k and ℓ, respectively, are the same surface.

Answer:

(c) There is a C^2 function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\frac{\partial f}{\partial x}(x, y) = xy = \frac{\partial f}{\partial y}(x, y).$$

Answer:

(d) There is a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$ such that the directional derivative $D_{\mathbf{u}} f(\mathbf{0}) > 0$ for every unit vector $\mathbf{u} \in \mathbb{R}^n$.

Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This problem has **four** parts.)

(a) For a function $f(\theta)$, the polar graphs of $r = f(\theta)$ and $r = f(-\theta)$ are different.

Answer:

(b) For $a \geq 0$, $\lim_{(x,y)\to(0,0)} \frac{|x|^a y^3 + x^2 y}{x^2 + y^2}$ exists.

Answer:

(c) For a C^2 function $f : \mathbb{R}^2 \to \mathbb{R}$ where $x = x(t)$ and $y = y(t)$ are each twice-differentiable functions of a variable t,

$$\frac{d^2 f}{dt^2} = \frac{\partial^2 f}{\partial x^2}\frac{d^2 x}{dt^2} + \frac{\partial^2 f}{\partial y^2}\frac{d^2 y}{dt^2}.$$

Answer:

(d) For a point (a, b) in $\mathbb{R}^2$, the tangent plane to the sphere $x^2 + y^2 + z^2 = 1$ at $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ is parallel to the tangent plane to the graph of $f(x, y) = -xy^2 - x + 2y$ at $(a, b, f(a, b))$.

Answer:

3. Consider the function $F : \mathbb{R}^3 \to \mathbb{R}$ given by $F(x, y, z) = x^2 + y^2 + 2\sqrt{2}xz$. For which numbers k does $F(x, y, z) = k$ describe a two-sheeted hyperboloid?

4. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be the function defined by

$$f(x,y,z) = \begin{cases} (x^2+y^2+z^2)\sin\left(\frac{1}{x^2+y^2+z^2}\right) & (x,y,z) \neq (0,0,0) \\ k & (x,y,z) = (0,0,0). \end{cases}$$

Find a value of k which makes f continuous at $(0,0,0)$.

5. Find a linear approximation to the function $\mathbf{g}(x, y, z) = (2^{x+y+z}, \sin(x + y - 2z))$ at $(1, 1, 1)$ and use it to approximate $\mathbf{g}(1, 0.9, 1.1)$. (Recall that the derivative of $f(x) = 2^x$ with respect to x is $2^x \ln 2$.)

6. (This problem has **two** parts.) The function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \frac{1}{69\pi} \sin\left((3x^2 + 5y^2)\pi\right) - 7$$

describes the air temperature in degrees Celsius on a patch of ice at position (x, y). Wally the Walrus is wallowing in some snow at position $(2, -1)$.

(a) In which direction should Wally waddle to warm up most quickly? Give your answer as a (not necessarily unit) vector.

(b) At some time, Wally waddles through the point $(3, 2)$ following the curve with parametric equations

$$(x(t), y(t)) = (t + 2, 3t^2 - 1),$$

where t is measured in hours. What is the rate of change in air temperature with respect to time that Wally experiences as he waddles through the position $(3, 2)$? The air temperature is described by the same function $f(x, y) = \frac{1}{69\pi} \sin\left((3x^2 + 5y^2)\pi\right) - 7$ as before.