Math 290-2: Linear Algebra & Multivariable Calculus
Northwestern University, Winter 2015

Course Information

- Instructor: Santiago Cañez
- Email: scanez@northwestern.edu
- Website: https://canvas.northwestern.edu/courses/14155/
- Office Hours: Tu 9:30–11:30am and Th 3–5pm in Lunt B27, or by appointment
- Lecture: MWF 12–12:50pm in Fisk B17
- Discussion: Tu 12–12:50pm in Fisk B17 with Angelo Lee
- Textbook: *Linear Algebra with Applications, 5th ed.* by Bretscher, and *Vector Calculus, 4th ed.* by Colley
- Prerequisites: Math 290-1 or instructor consent

Topics Covered

Orthonormal bases, orthogonal and symmetric matrices, curves and surfaces, multivariable limits and derivatives, directional derivatives and gradients, extrema and Lagrange multipliers

What Is This Course About?

This quarter, after quickly finishing up our study of linear algebra, we begin our study of multivariable calculus. As the name suggests, multivariable calculus is all about doing calculus with functions of more than one variable, such as the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = x^2 y.$$

This functions takes as input *two* numbers x and y and spits out the number x^2y. Working with single-variable functions is all well and good for a first calculus course, but of course most things in life actually depend on multiple parameters (variables), and so developing calculus for such functions is crucial in applications.

At a basic level, I claim you already know how to do calculus with multivariable functions, simply using what you know about single-variable functions as a guide. For instance, take a wild guess as to what the following means: "Compute the derivative of the function $f(x, y) = x^2y$ with respect to x". The "with respect to x" part is key: when differentiating with respect to x we treat anything else, such as y, like a constant so that our function above looks like $f(x, y) = x^2(\text{constant})$. Differentiating with respect to our variable x then gives $2xy$ in this case. Similarly, "compute the derivative of f with respect to y" means to treat x as a constant and take the usual derivative as if y were the only variable—this would give x^2 as a result. These are what are known as the *partial derivatives* of f, and much time will be spent on understanding what these quantities mean.

However, what on earth could we possibly mean by asking for *the* derivative of f without reference to one of the variables? A multivariable function will have many partial derivatives, but we would still like to speak about the derivative of f as an object itself. Here is where the connection to linear algebra crops up: *the* derivative of a function $f : \mathbb{R}^2 \to \mathbb{R}$ will turn out to be a 1×2 matrix, and in general the derivative of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ will be an $m \times n$ matrix. So, in higher dimensions there is a deep connection between derivatives and matrices; this wasn't noticeable in single-variable calculus since the derivative of a function $f : \mathbb{R} \to \mathbb{R}$ is a 1×1 matrix,

and 1×1 matrices aren't interesting. So, we will see that much of the linear algebra we developed will indeed help us understand what derivatives mean in higher dimensions. In particular, we'll see that eigenvalues and eigenvectors are the correct tools needed to classify local extrema points (minimums, maximums, etc.) in higher dimensions.

More so than our class last quarter, this course will be extremely geometric. Of course, we all know usual derivatives are related to geometry in that they tells us something about slopes of tangent lines, and the same geometric intuitions which show up in single-variable calculus will show up here. By the time we are finished, you too will be experts at drawing and visualizing curves and surfaces in 3 dimensions. This is crucial, since calculus isn't just about manipulating some formulas and expressions, but rather it has deeply geometric roots.

What Should You Already Know?

Of course, you should have taken Math 290-1 or a similar linear algebra course. We will spend the first three weeks finishing off linear algebra, so expect to jump right back into things right away. These first three weeks are also a good time to start reviewing some calculus; in particular, you should be able to compute (single-variable) derivatives with ease using the product rule, chain rule, etc. There's no need to review integration techniques until next quarter.

Homework, Quizzes, and Exams

There will be weekly homework assignments due on Fridays. You are encouraged to work together on problem sets, but each of you must hand in your own work in your own writing. There will also be weekly quizzes, which you'll take in discussion section on Tuesdays. In the end your lowest homework and quiz scores will be dropped.

There will be two midterms and a final exam. The midterms will be held on February 2nd and March 2nd—both Mondays—from 6:30–7:30pm in a room to be determined. The final will be held on Monday, March 16th from 12–2pm. Please see me as soon as possible if you have a conflict.

Grades

Your final score will be composed of homework, quiz, and exam scores according to the following percentages: 10% Quizzes, 15% Homework, 20% Midterm 1, 20% Midterm 2, 35% Final Exam. What constitutes an A, B, etc. will be determined at the end once all scores have been totaled, so there is no set scale. However, I'll try to give a sense of where you stand throughout the quarter.

University Policies

Students are required to abide by Northwestern University's academic integrity policy, which can be found at http://www.northwestern.edu/provost/students/integrity/. Failure to adhere to this policy will likely result in a failing grade in the class and/or expulsion from the University.

Any student requesting accommodations related to a disability or other condition is required to register with AccessibleNU (847-467-5530) and provide professors with an accommodation notification from AccessibleNU, preferably within the first two weeks of class. All information will remain confidential.