Math 290-3: Linear Algebra & Multivariable Calculus
Northwestern University, Spring 2015

Course Information

- Instructor: Santiago Cañez
- Email: scanez@northwestern.edu
- Website: https://canvas.northwestern.edu/courses/20724
- Office Hours: TBD in Lunt B27
- Lecture: Section 61 MWF 12–12:50pm in Lunt 105
- Discussion: Section 61 Tu 12–12:50pm in Lunt 105 with Rebecca Wei
- Textbook: *Vector Calculus, 4th ed.* by Colley
- Prerequisites: Math 290-2 or Math 291-2

Topics Covered

Areas and Volumes, Double and Triple Integrals, Change of Variables, Vector Fields and Line Integrals, Green's Theorem, Surface Integrals, Stokes' Theorem and Gauss's Theorem

What Is This Course About?

This final quarter of our year-long sequence is all about *integration*, specifically integration of multivariable functions and integration of vector fields. What is a vector field and what does it mean to integrate one? We'll see the answer to this in the final few weeks of the quarter, where much of the multivariable calculus we've done so far will all come together in a magnificent blaze of mathematical glory.

At the beginning of last quarter I said that you all essentially already knew how to compute multivariable derivatives, since partial differentiation boils down to computing a good old-fashioned derivative after holding other variables constant. In a similar manner, I claim you already know how to compute a so-called *double integral* such as

$$\int_0^1 \int_0^1 xy \, dy \, dx.$$

As the notation suggests, you first compute the "inside" integral with respect to y thinking of x as constant, and then compute the "outside" integral. However, as with derivatives, there is a new twist in that now the *bounds* of integration no longer need to be constant; for instance, what on earth does

$$\int_0^1 \int_{-x}^{x} xy \, dy \, dx$$

mean? The tricky part of multivariable integration comes in dealing with these bounds, and setting these up will require a good geometric understanding of what all these things are supposed to represent. We'll see polar, cylindrical, and spherical coordinates again, and hopefully gain a better appreciation as to why these are so useful. In addition, our good friend—the determinant and its geometric interpretation—will make an appearance when discussing what happens when you make a general "change of variables" in a multiple integral.

This all builds up to what is commonly known as *vector calculus*, which is an in depth study of vector fields and their applications. A vector field is a way to talk about the "flow" of something,

whether it be the flow of water in a lake, the flow of electrons in a bolt of lightning, or something more abstract like the flow of money or commodities from one sector of an economy to another. Imagining a vector field as literally describing the flow of a liquid will give the best intuition. We'll be interested in two main questions: given some path along which we'll move, how hard/easy is it to do so when we have a vector field either pushing "against" or "with" us, and given a surface placed within the vector field, how can we measure the extent to which the vector field is flowing "through" the surface? This will lead us to the concepts of line and surface integrals, and it is here that we will see far-reaching and amazing generalizations of the Fundamental Theorem of Calculus, expressing a deep connection between integration and differentiation. Good stuff.

What Should You Already Know?

Certainly, you should have taken Math 290-2 or a similar course. You should be comfortable with multivariable differentiation and related topics. It might also be a good idea to review some single-variable integration topics: how to compute integrals of standard functions, substitution, integration by parts, etc.

Homework, Quizzes, and Exams

There will be weekly homework assignments, usually due on Fridays. You are encouraged to work together on problem sets, but each of you must hand in your own work in your own writing. There will also be weekly quizzes, which you'll take in discussion section on Tuesdays. In the end your lowest homework and quiz scores will be dropped.

There will be two midterms and a final exam. The midterms will be held on Thursday, April 30th and Thursday, May 21st from 6:30–7:30pm in a room to be determined. The final will be held on Tuesday, June 9th from 9–11am. Please see me as soon as possible if you have a conflict.

Grades

Your final score will be composed of homework, quiz, and exam scores according to the following percentages: 10% Homework, 15% Quizzes, 20% Midterm 1, 20% Midterm 2, 35% Final Exam. What constitutes an A, B, etc. will be determined at the end once all scores have been totaled, so there is no set scale. However, I'll try to give a sense of where you stand throughout the quarter.

University Policies

Students are required to abide by Northwestern University's academic integrity policy, which can be found at http://www.northwestern.edu/provost/students/integrity/. Failure to adhere to this policy will likely result in a failing grade in the class and/or expulsion from the University.

Any student requesting accommodations related to a disability or other condition is required to register with AccessibleNU (847-467-5530) and provide professors with an accommodation notification from AccessibleNU, preferably within the first two weeks of class. All information will remain confidential.