Math 291-3: Final Exam
Northwestern University, Spring 2017

Name: ____________________

1. (15 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If $f(x, y)$ is continuous everywhere except on the set of points satisfying $x^2 + 2y^2 \leq 1$, then f is integrable over the rectangle $[-3, 3] \times [-3, 3]$.

(b) If $\mathbf{F}$ is C^1 on an open set $U \subseteq \mathbb{R}^2$ and $\operatorname{curl} \mathbf{F} = \mathbf{0}$ on U, then $\mathbf{F}$ is conservative on U.

(c) If S_1 and S_2 are oriented surfaces with the same boundary and which induce the same orientation on that boundary, then $\iint_{S_1} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_2} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ for any C^1 vector field $\mathbf{F}$.

Problem	Score
1	
2	
3	
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Total	

2. (10 points) Consider the following iterated integral:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{-1}^{-\sqrt{x^2+y^2}} y^2 \, dz \, dx \, dy.$$

(a) Rewrite this as an iterated integral in cylindrical coordinates.
(b) Rewrite this as an iterated integral in spherical coordinates.

3. (10 points) Suppose A is an invertible $n \times n$ matrix. Let D be a compact region in $\mathbb{R}^n$ such that the constant function 1 is integrable over D and let $A(D)$ denote the image of D under the linear transformation $\mathbb{R}^n \to \mathbb{R}^n$ defined by A. Show that

$$\operatorname{Vol} A(D) = |\det A| \operatorname{Vol}(D).$$

(This is the geometric interpretation of the determinant as an expansion factor we first introduced last quarter, but the point is to justify this using material from this quarter.)

4. (10 points) Let C be the curve where the cylinder $y^2 + z^2 = 1$ and the plane $x = y$ intersect. Show that C is smooth everywhere. (Recall that a curve is said to be *smooth* at the points where its tangent vector is nonzero.)

5. (10 points) Show that a C^1 vector field $\mathbf{F}$ on $\mathbb{R}^n$ has path-independent line integrals if and only if its line integral over any closed smooth C^1 curve is zero. (Recall that $\mathbf{F}$ having path-independent line integrals means that if C_1 and C_2 are smooth C^1 curves which begin at the same point and end at the same point, then $\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = \int_{C_2} \mathbf{F} \cdot d\mathbf{s}$. You may not use the fact that both of these properties are equivalent to $\mathbf{F}$ being conservative.)

6. (10 points) Compute the line integral

$$\int_C (x \sin e^x - xz)\, dx - 2xy\, dy + (z^2 + y)\, dz$$

where C is the curve consisting of the line segment from $(2, 0, 0)$ to $(0, 2, 0)$, followed by the line segment from $(0, 2, 0)$ to $(0, 0, 2)$, followed by the line segment from $(0, 0, 2)$ to $(2, 0, 0)$. Hint: C lies on the plane $x + y + z = 2$.

7. (10 points) Compute the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F} = (3x - ye^{\cos z})\,\mathbf{i} + (e^{x^{10}z^8} - 2yz)\,\mathbf{j} + (z^2 + ye^x)\,\mathbf{k}$$

where S is the portion of the cylinder $x^2 + y^2 = 1$ which lies between $z = 0$ and $z = 1$, oriented with inward pointing normal vectors.