Math 291-3: Midterm 1
Northwestern University, Spring 2016

Name: ______________________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) Any Riemann sum of the function $f(x,y) = x$ over $[-1,1]\times[-1,1]$ is positive.

(b) The function

$$f(x,y) = \begin{cases} x^2+y^2 & \|(x,y)\| \le 1 \\ \frac{2}{x^2+y^2} & \|(x,y)\| > 1 \end{cases}$$

is integrable over the square $[-1,1]\times[-1,1]$.

Problem	Score
1	
2	
3	
4	
5	
Total	

$n \geq 3$ with vertices on a given circle

2. (10 points) An n-sided polygon with side lengths $x_1, \ldots, x_n$ has area

$$A = \sqrt{(s - x_1)(s - x_2) \cdots (s - x_n)}$$ missing s^{4-n}

where s is half the perimeter. Show that among all n-sided polygons with fixed perimeter P, there is one with maximal area and determine the lengths of all its sides.

and vertices on a given circle

Just ignore area interpretation to get a meaningful problem.

3. (10 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $f(x, y) = -x^2 - y^3 + 3y$ and let $D \subseteq \mathbb{R}^2$ be the region enclosed by the circle $x^2 + y^2 = 2y$. Show that

$$\iint_D f(x, y)\, dA \leq 2\pi.$$

Hint: You can take it for granted without justification that the maximum value of f over D does not occur on the boundary of D.

4. (10 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a continuous function. Rewrite the following as an iterated integral with respect to the order $dy\,dz\,dx$.

$$\int_0^1 \int_0^{1-y} \int_0^{y^2} f(x, y, z)\, dz\, dx\, dy.$$

5. (10 points) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying

$$\int_0^1 (1-x)f(x)\,dx = 5.$$

Find the value of the double integral

$$\int_0^1 \int_0^x f(x-y)\,dy\,dx.$$

Hint: Let $u = x - y$ and use this as one of the new variables in a suitable change of variables application.