Math 291-3: Midterm 1
Northwestern University, Spring 2017

Name: ____________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If $f : [-1, 1] \times [-2, 2] \times [-3, 3] \to \mathbb{R}$ is a constant function, then all Riemann sums of f (for any partition of $[-1, 1] \times [-2, 2] \times [-3, 3]$ and any collection of sample points) have the same value.

(b) If $f : [-5, 5] \times [-5, 5] \to \mathbb{R}$ is bounded but not continuous, then f is not integrable.

Problem	Score
1	
2	
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2. (10 points) Fix $K > 0$ and consider all nonnegative numbers $x_1, \ldots, x_n$ satisfying

$$x_1 + x_2 + \cdots + x_n = K.$$

Show that among all such numbers there exists ones which maximize the product $x_1 x_2 \cdots x_n$ and find the specific values of those which do.

3. (10 points) Show that for any compact region $D \subseteq \mathbb{R}^2$ of area 10, the following inequality holds:

$$\iint_D (3 - x^2 + 2x - y^2 + 2y)\, dA \leq 50.$$

You may assume that any local maximum of $f(x, y) = 3 - x^2 + 2x - y^2 + 2y$ is actually a global maximum.

4. (10 points) Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is continuous. Rewrite the following as an iterated integral with respect to the order $dy\,dx\,dz$.

$$\int_0^1 \int_{z^2}^1 \int_0^{1-y} f(x, y, z)\, dx\, dy\, dz$$

5. (10 points) Write the following as a single iterated integral in polar coordinates.

$$\int_0^1 \int_y^1 (x^2 + y^2)\, dx\, dy + \int_1^2 \int_0^{\sqrt{2x - x^2}} (x^2 + y^2)\, dy\, dx$$

Note that the order of integration in the first expression is $dx\, dy$ while in the second it is $dy\, dx$.