Math 291-2: Midterm 1
Northwestern University, Winter 2017

Name: ______________________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) A 3×3 matrix with determinant 1 must be orthogonal.

(b) If λ is a real eigenvalue of an orthogonal matrix, then $\lambda = \pm 1$.

Problem	Score
1	
2	
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Total	

2. (10 points) Suppose $\mathbf{u}_1, \ldots, \mathbf{u}_n$ are orthonormal vectors in $\mathbb{R}^n$. Show that for any $\mathbf{x} \in \mathbb{R}^n$,

$$\mathbf{x} = (\mathbf{x} \cdot \mathbf{u}_1)\mathbf{u}_1 + \cdots + (\mathbf{x} \cdot \mathbf{u}_n)\mathbf{u}_n.$$

3. (10 points) Find two 3×3 orthogonal matrices Q satisfying

$$Q \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

4. (10 points) Suppose n is odd and that A is an $n \times n$ matrix which is skew-symmetric, meaning $A^T = -A$. Show that A is not invertible. Hint: What is the determinant of A?

5. (10 points) Let $T : P_6(\mathbb{R}) \to P_6(\mathbb{R})$ be the linear transformation which sends $p(x)$ to $p(-x)$. (To be clear, $p(-x)$ is the polynomial you get by replacing with $-x$ all instances of x in $p(x)$.) Determine the eigenvalues of T and find a basis for each of its eigenspaces.