Math 291-2: Midterm 1
Northwestern University, Winter 2018

Name: ______________________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If $\mathbf{v}_1, \mathbf{v}_2$ is a basis of $\mathbb{R}^2$ and $\mathbf{x} = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$, then $\mathbf{x} = \operatorname{proj}_{\mathbf{v}_1}\mathbf{x} + \operatorname{proj}_{\mathbf{v}_2}\mathbf{x}$.

(b) If A is a 2×2 matrix which sends a disk of radius 2 onto a disk of radius 1, then $|\det A| < 1$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Let A be an $n \times n$ symmetric matrix and let V be a subspace of $\mathbb{R}^n$ with the property that $A\mathbf{v} \in V$ for any $\mathbf{v} \in V$. Show that if $\mathbf{w} \in V^\perp$, then $A\mathbf{w} \in V^\perp$.

3. (10 points) Suppose A and B are $n \times n$ orthogonal matrices such that AB^T is upper triangular with positive diagonal entries. Show that $A = B$. Hint: The product of orthogonal matrices is orthogonal.

4. (10 points) Suppose A, B are $n \times n$ matrices. Show that $\det(AB) = (\det A)(\det B)$. Hint: In the case where A is invertible, consider what happens when you row-reduce the matrix $\begin{bmatrix} A & AB \end{bmatrix}$ to turn the A on the left into I.

5. (10 points) Let $T : P_5(\mathbb{R}) \to P_5(\mathbb{R})$ be the linear transformation defined by

$$T(p(x)) = 2x^2 p''(x).$$

Determine all eigenvalues and eigenvectors of T. Be sure to justify why the eigenvalues and eigenvectors you find are indeed all of them.