Math 291-1: Midterm 2
Northwestern University, Fall 2015

Name: ____________________

1. (10 points) Determine, with justification, whether each of the following is true or false.
(a) There exists a 2×2 non-identity matrix A such that $A^5 = I$.
(b) The space of 3×3 upper-triangular complex matrices has a 7-dimensional complex subspace.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that

$$T\begin{pmatrix} 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \text{ and } T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and that $S : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation such that

$$S\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } S\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If A denotes the standard matrix of the composition ST, compute A^2 explicitly and explain why there is not enough information to determine the standard matrix of TS.

3. (10 points) Suppose $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^2$ are linearly independent and that A is a 2×2 matrix such that $A\mathbf{v}_1 = \mathbf{v}_2$ and $A\mathbf{v}_2 = \mathbf{v}_1$. Show that the columns of A span $\mathbb{R}^2$.

Bonus (1 extra point): If $\mathbf{v}_1$ and $\mathbf{v}_2$ have the same length, show that A describes a reflection.

4. (10 points) Let A be an $n \times n$ matrix and suppose that $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ satisfy

$$A\mathbf{x} = \mathbf{x} \quad \text{and} \quad A\mathbf{y} = 3\mathbf{y}.$$

Also, let U be a subspace of $\mathbb{R}^n$ with the property that if $\mathbf{u} \in U$, then $A\mathbf{u} \in U$ as well. If $\mathbf{x}+\mathbf{y} \in U$, show that $\mathbf{x} \in U$ and $\mathbf{y} \in U$.

5. (10 points) Let W be the subspace of $P_3(\mathbb{C})$ consisting of the polynomials $p(x) \in P_3(\mathbb{C})$ which satisfy

$$p(x) + p(-x) = 0.$$

Find a basis for W considered as a vector space over $\mathbb{R}$.