Math 291-3: Midterm 2
Northwestern University, Spring 2017

Name: ______________________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If $\mathbf{F}$ is C^1 and satisfies $\operatorname{div}\mathbf{F} = x$, then there does not exist a C^2 field $\mathbf{G}$ such that $\operatorname{curl}\mathbf{G} = \mathbf{F}$.

(b) If C is a curve and $\int_C \mathbf{F} \cdot d\mathbf{s} = 0$, then $\mathbf{F}$ is conservative.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Recall that the surface area of a smooth C^1 surface with parametrization $\mathbf{X}(u, v)$ where $(u, v) \in D$ is given by

$$\iint_D \|\mathbf{X}_u \times \mathbf{X}_v\| \, du \, dv.$$

Compute the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ lying below $z = 4$.

3. (10 points) Suppose $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a C^2 vector field. Show that

$$\operatorname{curl}(\operatorname{curl}\mathbf{F}) = \nabla(\operatorname{div}\mathbf{F}) - \langle \operatorname{div}(\nabla P), \operatorname{div}(\nabla Q), \operatorname{div}(\nabla R)\rangle.$$

Start by computing the left-hand side.

4. (10 points) Suppose C is the curve consisting of the line segment from $(0,0)$ to $(1,2)$, followed by the line segment from $(1,2)$ to $(2,0)$. Compute the following line integral:

$$\int_C (2xye^{x^2y} + e^y)\,dx + x^2e^{x^2y}\,dy$$

5. (10 points) Suppose C is the ellipse $4x^2 + 9y^2 = 1$ oriented counterclockwise. Determine the value of the line integral

$$\int_C \frac{y\,dx - x\,dy}{x^2 + y^2},$$

justifying every step you take along the way. The only thing you may take for granted is that the exterior derivative of the 1-form in question is 0. Hint: Argue that you can replace C by a different curve.