Math 291-3: Midterm 2
Northwestern University, Spring 2018

Name: ______________________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If $U \subseteq \mathbb{R}^2$ is open and $\mathbf{F}$ is a C^1 vector field of curl zero on U, then $\mathbf{F}$ is conservative on U.

(b) There does not exist a 1-form ω on $\mathbb{R}^3$ such that $d\omega = x\,dy \wedge dz + y\,dz \wedge dx + z\,dx \wedge dy$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Suppose $\mathbf{F}, \mathbf{G}$ are C^1 vector fields on $\mathbb{R}^3$. Show that

$$\operatorname{div}(\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \operatorname{curl}(\mathbf{F}) - \mathbf{F} \cdot \operatorname{curl}(\mathbf{G})$$

where $\mathbf{F} \times \mathbf{G}$ denotes the vector field defined by $(\mathbf{F} \times \mathbf{G})(\mathbf{p}) = \mathbf{F}(\mathbf{p}) \times \mathbf{G}(\mathbf{p})$.

3. (10 points) Suppose $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ is a C^1 vector field on $\mathbb{R}^n$ and that C is an oriented smooth C^1 curve in $\mathbb{R}^n$. Show that the line integral of $\mathbf{F}$ over C is independent of parametrization of C. To be precise, if $\mathbf{x} : [a, b] \to \mathbb{R}^n$ is a smooth C^1 parametrization of C and $\mathbf{y} = \mathbf{x} \circ \tau$ is a different parametrization which determines the same orientation, where $\tau : [c, d] \to [a, b]$ is an injective, onto C^1 change of variables with nonzero derivative, you want to show that the line integral of $\mathbf{F}$ computed using $\mathbf{x}$ is the same as the line integral computed using $\mathbf{y}$.

4. (10 points) Suppose C_1, C_2 are smooth oriented C^1 curves in $\mathbb{R}^3$ which both begin at a point $\mathbf{q} \in \mathbb{R}^3$ and end at a point $\mathbf{p} \in \mathbb{R}^3$. Show that

$$\int_{C_1} \langle y^2z^3, 2xyz^3 + z, 3xy^2z^2 + y - z \rangle \cdot d\mathbf{s} = \int_{C_2} \langle y^2z^3, 2xyz^3 + z, 3xy^2z^2 + y - z \rangle \cdot d\mathbf{s}.$$

5. (10 points) Let C be the top half of the unit circle oriented counterclockwise. Compute

$$\int_C (y^2x + x^2)\,dx + (x^2y + x - y^{y^2 \sin^2 y + 1})\,dy.$$

Hint: Find a way to apply Green's Theorem.