Math 291-2: Midterm 2
Northwestern University, Winter 2017

Name: ____________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) There is a 2×2 symmetric matrix A such that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(b) There is a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $Df(\mathbf{x}) = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ for every $\mathbf{x} \in \mathbb{R}^2$.

Problem	Score
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2	
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2. (10 points) Determine the values of k for which the following matrix is diagonalizable. The eigenvalues are $k, 1$, and -3.

$$A = \begin{bmatrix} 0 & 0 & 3 \\ 3 & k & 3 \\ 1 & 0 & -2 \end{bmatrix}$$

3. (10 points) For fixed k, determine whether the surface in $\mathbb{R}^3$ with equation

$$3x^2 - y^2 + 3z^2 + 2xz = k$$

is an ellipsoid, a double cone, a one-sheeted hyperboloid, or a two-sheeted hyperboloid. The answer will depend on k.

4. (10 points) Recall that $U \subseteq \mathbb{R}^2$ is open if for any $\mathbf{p} \in U$, there exists $r > 0$ such that $B_r(\mathbf{p}) \subseteq U$. Show, using this definition, that the region

$$\{(x, y) \in \mathbb{R}^2 \mid -1 < x < 1, -1 < y < 1\}$$

is open. (This is the square with vertices at $(-1,-1), (-1,1), (1,-1)$, and $(1,1)$, only with the corners and sides of the square excluded.)

5. (10 points) Determine whether or not the following function is differentiable at $(0, 0)$.

$$f(x, y) = \begin{cases} y - \frac{3x^3 - 2y^4}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$