Math 291-2: Midterm 2
Northwestern University, Winter 2018

Name: ______________________________

1. (10 points) Determine whether each of the following statements is true or false. If it is true, explain why; if it is false, give a counterexample.

(a) If A is a symmetric matrix whose only eigenvalue is 3, then $A = 3I$.

(b) The level surfaces of $f(x, y, z) = x^2 - y^2 - z^2$ are all hyperboloids.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Let A be a 5×5 matrix of rank 3, for which 1 is an eigenvalue with a 1-dimensional eigenspace and -1 an eigenvalue with a 2-dimensional eigenspace. Show that $A^5 = A$. Hint: It is not true that 1 and -1 are the only eigenvalues of A.

3. (10 points) Find the two points on the surface given by the following equation which are closest to the origin.

$$-x^2 + y^2 - z^2 + 10xz = 1$$

To save some work, take it for granted that the eigenvalues of $\begin{bmatrix} -1 & 0 & 5 \\ 0 & 1 & 0 \\ 5 & 0 & -1 \end{bmatrix}$ are $1, -6, 4$.

4. (10 points) Show that the set $U \subseteq \mathbb{R}^3$ defined by $U = \{(x, y, z) \in \mathbb{R}^3 \mid y \neq 1\}$ is open in $\mathbb{R}^3$.

5. (10 points) Show that the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{-2x^3+3y^4}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

is not differentiable at $(0,0)$, even though both partial derivatives $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ exist.