Math 300: Midterm 1

Northwestern University, Spring 2017

Name: ______________________________

1. (10 points) Give an example of each of the following with brief justification.

(a) Sets S, A, B such that $S \subseteq A \cup B$ but $S \not\subseteq A$ and $S \not\subseteq B$.

(b) A subset A of $\mathbb{R}$ such that $(\mathbb{R} \times \mathbb{R}) - (A \times A) \neq (\mathbb{R} - A) \times (\mathbb{R} - A)$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Suppose A and B are the following sets:

$$A = \{n \in \mathbb{Z} \mid n = 6k - 4 \text{ for some } k \in \mathbb{Z}\} \text{ and}$$

$$B = \{n \in \mathbb{Z} \mid n = 3k + 2 \text{ for some } k \in \mathbb{Z}\}.$$

Show that $A \subseteq B$ and $B \not\subseteq A$.

3. (a) (5 points) Suppose $x \in \mathbb{R}$. Show that if $1 - r \leq x$ for all $r > 0$, then $1 \leq x$.
(b) (5 points) Determine the following intersection, and prove that your answer is correct.

$$\bigcap_{r>0} (1 - r, 3]\,.$$

To be clear, we are considering intervals $(1 - r, 3] = \{x \in \mathbb{R} \mid 1 - r < x \leq 3\}$ as r ranges over all positive real numbers.

4. (10 points) Suppose A, B, C are sets. Show that

$$(A \cap B) - C = (A \cap B) - (A \cap C).$$

5. (10 points) Suppose $n \in \mathbb{Z}$. Show that n is divisible by 10 if and only if n is divisible by both 2 and 5. (Recall that to say n is divisible by $k \in \mathbb{Z}$ means there exists $\ell \in \mathbb{Z}$ such that $n = k\ell$.)

It is NOT enough to say something along the lines of "if n has 2 and 5 as factors, then it must have $2 \cdot 5 = 10$ as a factor as well since 2 and 5 have no common factors apart from ± 1" without proof. Ask if you're unsure about whether you can take some fact for granted.