Math 300: Midterm 1
Northwestern University, Spring 2018

Name: ______________________________

1. Give an example of each of the following with brief justification.
(a) (5 points) A false implication (i.e. "if P, then Q" statement) whose converse is true.
(b) (5 points) Subsets A, B of $\mathbb{R}$ such that $\mathbb{R}\backslash(A \cup B)$ is the empty set.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Let $n \in \mathbb{Z}$. Show that 9 divides n if and only if 9 divides $4n$. You may use basic properties of even and odd integers (i.e. what happens when you multiply two odd integers together, two even integers together, or an odd with an even), but no other properties of relatively prime integers. For instance, saying something along the lines of "if $4n$ is divisible by 9, then n is divisible by 9 since 4 and 9 are relatively prime" is not enough.

3. (10 points) Let A, B, C be subsets of some larger set. Show that $A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash C)$.

4. At no point in either part below can you take some kind of limit. You must find a different way.

(a) (5 points) Let $x \in \mathbb{R}$. Show that if $x < 5 + \frac{1}{n^2}$ for all $n \in \mathbb{N}$, then $x \leq 5$.

(b) (5 points) Determine the following intersection and prove that your answer is correct.

$$\bigcap_{n \in \mathbb{N}} [-1, 5 + \tfrac{1}{n^2})$$

5. (10 points) Suppose $A \subseteq \mathbb{R}$ has a supremum, and let $4+A$ denote the set of all numbers obtained by adding 4 to elements of A:

$$4 + A := \{4 + a \mid a \in A\}.$$

Show that $\sup(4 + A) = 4 + \sup A$. (Note: $4 + A$ is simply notation we use to the describe the set in question; we are not literally adding the number 4 to the set A.)