Math 320-1: Midterm 1
Northwestern University, Fall 2014

Name: ____

1. (10 points) Give an example of each of the following. You do not have to justify your answer.

(a) A subset of $\mathbb{Q}$ with an irrational infimum and no supremum.

(b) A strictly decreasing sequence which converges to π.

(c) A Cauchy sequence (x_n) of nonzero numbers such that the sequence $\left(\frac{1}{x_n}\right)$ is not Cauchy.

(d) A nonconstant convergent sequence (x_n) and bounded sequence (y_n) such that $(x_n y_n)$ does not converge.

2. (10 points) Determine the supremum of

$$A = \left\{ \frac{n-1}{2n-1} \;\middle|\; n \in \mathbb{N} \right\}$$

and prove that your answer is correct.

3. (10 points) Suppose that (x_n) is a sequence which converges to -1. Show that the sequence $(\sqrt[3]{x_n})$ of cube roots also converges to -1. Hint: For any $a, b \in \mathbb{R}$, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

4. (10 points) Show that the sequence (x_n) defined by

$$x_n = (-1)^n \left(\cos(\sin n) - \frac{2\cos n}{n} + \frac{\sin n}{n^2} \right)$$

has a convergent subsequence.

5. (10 points) Suppose that (a_n) is a sequence of positive numbers such that the sequence (x_n) defined by

$$x_n = a_1 + a_2 + \cdots + a_n$$

converges. If (b_n) is a bounded sequence, show that the sequence (y_n) defined by

$$y_n = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

also converges.