Math 320-2: Midterm 1
Northwestern University, Winter 2015

Name: ______________________________

1. (10 points) Give an example of each of the following. You do not have to justify your answer.

(a) A series $\sum a_n$ of numbers which converges such that $\sum a_n^2$ does not converge.

(b) A sequence of functions on $(0, 1)$ which converges pointwise but not uniformly.

(c) A power series centered at 1 with radius of convergence 3.

(d) A function which is bounded and analytic on $\mathbb{R}$.

2. (10 points) Suppose that for each $n \in \mathbb{N}$, $f_n : [-2, 2] \to \mathbb{R}$ is an increasing function and that the sequence (f_n) converges pointwise to the constant function $f(x) = 1$. Show that $f_n \to f$ uniformly on $[-2, 2]$ as well. (Note: the assumption that each f_n is increasing is important.)

3. (10 points) Determine, with justification, the value of the following limit.

$$\lim_{n\to\infty} \int_0^1 \left[1 + \sin\left(2\cos\frac{x}{n} - 2\right)\right]\, dx$$

You may use the fact that $|\sin y| \leq |y|$ for all $y \in \mathbb{R}$ without proof.

4. (10 points) Show that the series

$$\sum_{n=1}^{\infty} \frac{ne^{x/n} - 1}{n^3 + 1}$$

converges uniformly on $(2, 4)$ to a differentiable function f such that $|f'(x)| \leq e^x$ for all $x \in (2, 4)$. You may use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^3+1} \leq 1$ without proof.

5. (10 points) Suppose that the series $\sum_{n=0}^{\infty} a_n(x-1)^n$ has radius of convergence $R > 0$. Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} n a_n x^{3n}.$$