## Math 320-1: Midterm 2 Northwestern University, Fall 2014

## Name:

- 1. (10 points) Give an example of each of the following. You do not have to justify your answer.
  - (a) A function f which is nowhere continuous but such that |f| is everywhere continuous.
  - (b) A continuous function on  $(4, \infty)$  which does not extend to a continuous function on  $[4, \infty)$ .
  - (c) A function f for which there does not exist a differentiable function F such that F' = f.
  - (d) A differentiable function f on (0, 1) such that f' is not uniformly continuous on (0, 1).

**2.** (10 points) Define  $f : \mathbb{R} \to \mathbb{R}$  to be the function

$$f(x) = \begin{cases} (x-1)\cos\frac{1}{x-1} & x > 1\\ x^2 - 1 & x \le 1 \text{ is rational}\\ 0 & x < 1 \text{ is irrational.} \end{cases}$$

Show that f is continuous at 1 but not at  $-\pi$ .

**3.** (10 points) Suppose that  $f, g : \mathbb{R} \to \mathbb{R}$  are bounded and uniformly continuous. Show that their product fg is also uniformly continuous. Hint: Rewrite f(x)g(x) - f(y)g(y) by adding and subtracting a common term.

**4.** (10 points) Suppose that  $f : \mathbb{R} \to \mathbb{R}$  is differentiable at 0 and satisfies f(0) = 0 and f'(0) < 0. Show that there exists b > 0 such that f is strictly negative on the interval (0, 2b). **5.** (10 points) Show that for any  $x \in [0, \frac{\pi}{4}]$ ,  $\sin x \le x \le \frac{\pi}{2} \sin x$ . Hint: Mean Value Theorem.