# Math 320-1: Midterm 2 Northwestern University, Fall 2014 

Name:

1. (10 points) Give an example of each of the following. You do not have to justify your answer.
(a) A function $f$ which is nowhere continuous but such that $|f|$ is everywhere continuous.
(b) A continuous function on $(4, \infty)$ which does not extend to a continuous function on $[4, \infty)$.
(c) A function $f$ for which there does not exist a differentiable function $F$ such that $F^{\prime}=f$.
(d) A differentiable function $f$ on $(0,1)$ such that $f^{\prime}$ is not uniformly continuous on $(0,1)$.
2. (10 points) Define $f: \mathbb{R} \rightarrow \mathbb{R}$ to be the function

$$
f(x)= \begin{cases}(x-1) \cos \frac{1}{x-1} & x>1 \\ x^{2}-1 & x \leq 1 \text { is rational } \\ 0 & x<1 \text { is irrational. }\end{cases}
$$

Show that $f$ is continuous at 1 but not at $-\pi$.
3. (10 points) Suppose that $f, g: \mathbb{R} \rightarrow \mathbb{R}$ are bounded and uniformly continuous. Show that their product $f g$ is also uniformly continuous. Hint: Rewrite $f(x) g(x)-f(y) g(y)$ by adding and subtracting a common term.
4. (10 points) Suppose that $f: \mathbb{R} \rightarrow \mathbb{R}$ is differentiable at 0 and satisfies $f(0)=0$ and $f^{\prime}(0)<0$. Show that there exists $b>0$ such that $f$ is strictly negative on the interval $(0,2 b)$.
5. (10 points) Show that for any $x \in\left[0, \frac{\pi}{4}\right], \sin x \leq x \leq \frac{\pi}{2} \sin x$. Hint: Mean Value Theorem.

