# Math 320-1: Real Analysis
## Northwestern University, Fall 2014

### Course Information

- Instructor: Santiago Cañez
- Email: scanez@northwestern.edu
- Website: `https://northwestern.instructure.com/courses/1926/`
- Office Hours: Tu 12-1pm, W 2-3pm, and Th 2-3pm in Lunt B27, or by appointment
- Lecture: MWF 1–1:50pm in Locy 214
- Discussion: Th 1–1:50pm in Locy 214 with Spencer Liang
- Textbook: *An Introduction to Analysis, 4th ed.* by Wade
- Prerequisites: Math 300 or instructor consent

### Topics Covered

Real Numbers and Completeness, Sequences and Limits, Continuity and Differentiation, Mean Value Theorem, Riemann Integration and Riemann Sums, Fundamental Theorem of Calculus

### What Is This Course About?

This course is an introduction to *real analysis*, or How I Learned to Stop Worrying and Love Inequalities. (Kudos to those of you who get the reference.) Real analysis is, essentially, the study of functions of real numbers, with notions such continuity, differentiability, and integrability being the main properties of functions we'll be interested in. At other schools such a course might instead be called "Advanced Calculus", and indeed this course is all about the theoretical foundations behind everything you ever did in a previous calculus course: understanding why things work the way they do and understanding why one should care. However, don't let the phrase "Advanced Calculus" fool you: in calculus you likely focused heavily on computations and applications and "advanced calculus" might suggest a more advanced version of the same, whereas we will mainly be interested in the theory of it all. Of course, the applications are still there (I hope to mention some of these as we go on), but the types of applications (say in statistics, economics, physics, computer science, and various other fields) in which this subject matter will be most useful require a firm understanding of theory.

Here is a simple motivating question, the answer to which you've implicitly had to use numerous times before in your mathematical lives but have probably never really thought hard about: given a continuous function from an interval $[a, b]$ to the set of real numbers, is it true that it must have a maximum and minimum? After all, if for some reason you want to maximize or minimize a function, you'd better know that such max and mins actually exist. Of course, you've had to find such max and mins on countless exams in previous calculus courses before, but here we are more interested in the question as to why this is even possible. In turns out that there is a property which the interval $[a, b]$ possesses (called *compactness*) which guarantees that any continuous function on it indeed has a maximum and a minimum. This is an important property—for instance, it is not true that every continuous function on the interval $(a, b)$ has a maximum and minimum. What is so different about the open interval $(a, b)$ as opposed to the closed interval $[a, b]$ that accounts for this? And what role does the continuity of a function play in all this? We will see the answer to this, and much, much more.

Here's a second motivating question: What is so "fundamental" about the Fundamental Theorem of Calculus? This is the result which says you can compute an integral by computing an antiderivative and then "evaluating at the endpoints", which is again something you've done hundreds of times before in previous courses. However, if you actually look at the precise definition of an integral you'll see that it has absolutely nothing to do with derivatives. The fact that integration and differentiation are intimately related is so important that it made Newton and Leibniz famous. Without this theorem, we would lack an "easy" way to compute integrals and the applications which this procedure depends on would collapse. Proving this Fundamental Theorem will be one of our end goals and will bring together much of what makes real analysis so powerful.

## What Should You Already Know?

The only official prerequisite is Math 300 or a similar course, which means you should be familiar with basic concepts from logic and set theory, and terminology involving functions and their properties. However, I think this is all possible to pick up along the way if you haven't taken Math 300 or if it's been a while since you did. Of course, you should have taken a calculus course before and be familiar with the notions of limits, derivatives, and integrals. But, since we'll be taking a more in-depth look at these topics there's no need to go back and practice differentiation and integration techniques—such computations won't be our focus.

## Homework and Exams

There will be weekly homework assignments, usually due on Fridays. You are encouraged to work together on problem sets, but each of you must hand in your own work in your own writing. Problems on assignments will almost always involve coming up with some type of proof, so a side goal of this course is to develop this skill further. There will also be weekly quizzes held in discussion on Thursdays, and in the end your lowest homework and quiz scores will be dropped.

There will be two midterms and a final exam. The midterms will be held in class on October 24th and November 14th, both Fridays. The final will be held on Thursday, December 11th from 9–11am. Please see me as soon as possible if you have a conflict.

## Grades

Your final score will be composed of homework and exam scores according to the following percentages: 10% Quizzes, 20% Homework, 15% min{Midterm 1, Midterm 2}, 25% max{Midterm 1, Midterm 2}, 30% Final Exam. What constitutes an A, B, etc. will be determined at the end once all scores have been totaled, so there is no set scale. However, I'll try to give a sense of where you stand throughout the quarter.

## University Policies

Students are required to abide by Northwestern University's academic integrity policy, which can be found at http://www.northwestern.edu/provost/students/integrity/. Failure to adhere to this policy will likely result in a failing grade in the class and/or expulsion from the University.

Any student with a disability requesting accommodations is required to register with Services for Students with Disabilities (ssd@northwestern.edu; 847-467-5530) and present an accommodation letter from SSD to his/her professor, preferably within the first two weeks of class. All information will remain confidential.