Math 320-2: Real Analysis
Northwestern University, Winter 2015

Course Information

- Instructor: Santiago Cañez
- Email: scanez@northwestern.edu
- Website: `https://canvas.northwestern.edu/courses/9533/`
- Office Hours: Tu 11:30am–12:30pm and Thursday 1–3pm in Lunt B27, or by appointment
- Lecture: MWF 1–1:50pm in Fisk B17
- Discussion: Tu 1–1:50pm in Lunt 101 with Xavier Garcia, or
Tu 1–1:50pm in Fisk B17 with Joel Specter
- Textbook: *An Introduction to Analysis, 4th ed.* by Wade
- Prerequisites: Math 320-1 or instructor consent

Topics Covered

Sequences and series of functions, uniform convergence, analytic functions, metric spaces and topology of $\mathbb{R}^n$, compact and connected sets, continuous functions and the Stone-Weierstrass Theorem

What Is This Course About?

This quarter of real analysis is mainly about generalizing some things we did last quarter to other settings. In particular, we will begin by looking at *sequences of functions* and what it means for such things to converge, and will then move on to studying so-called *metric spaces*. The point is that many things we saw previously for sequences of real numbers and continuous functions on $\mathbb{R}$ hold in much more general settings, and by looking at these generalizations we can start to put various concepts in the proper context.

A *metric space* is a "space" on which we have a notion of "distance" between elements. If you recall, most definitions we saw last quarter were phrased in terms of expressions of the form $|x-y|$, which gives the distance between x and y in $\mathbb{R}$. If you take these same definitions and replace $|x-y|$ by a more general type of distance, you get notions of convergent sequences and continuous functions in more general types of spaces; in particular, higher-dimensional spaces and "spaces" whose "points" are functions. In the second half of this quarter will see some overlap between topics we will cover and topics we covered last quarter, only now from this more general metric space setting.

Here is one application we will come to eventually, which sheds some light on why the generalizations mentioned above will prove useful. Consider an ordinary differential equation such as

$$f'(x) = 3|f(x)| - \log(e^{\sin\cos x})$$

with initial condition $f(1) = 1$. To solve this equation means to find the differentiable function satisfying the given conditions. In this case, it is not possible to find the solution by hand, and yet I claim that such a function must exist. The key in understanding why is to rephrase this differential equation in another way by taking integrals: the function f we want must satisfy

$$f(x) = 1 + \int_1^x (3|f(t)| - \log(e^{\sin\cos t}))\,dt,$$

as a consequence of the Fundamental Theorem of Calculus. Now, consider a "function" T which takes as input a function f and outputs the function given by the right side of the equation above; so, this is a "function" from a "space" of functions to a space of functions. To solve the original differential equation then means to find a function such that $T(f) = f$, and low-and-behold it turns out that properties of continuous functions we will consider for metric spaces will guarantee that such a function exists. At this point don't worry if any of this makes any sense—the upshot is that a calculus question gets rephrased as a metric space question, and we will be able to apply the metric space machinery we'll learn.

What Should You Already Know?

You should have taken Math 320-1 or a similar course and be familiar with the basics of analysis on $\mathbb{R}$, including topics such as sequences of real numbers, continuous functions on $\mathbb{R}$, differentiability, and integrability. If it's been awhile since you took such a course or you're transferring in from 321-1, let me know so that I can give you suggestions about what you should review.

Homework and Exams

There will be weekly homework assignments, usually due on Fridays. You are encouraged to work together on problem sets, but each of you must hand in your own work in your own writing. Problems on assignments will almost always involve coming up with some type of proof, so a side goal of this course is to develop this skill further. There will also be weekly quizzes held in discussion on Tuesdays, and in the end your lowest homework and quiz scores will be dropped.

There will be two midterms and a final exam. The midterms will be held in class on February 6th and February 27th, both Fridays. The final will be held on Friday, March 20th from 9–11am. Please see me as soon as possible if you have a conflict.

Grades

Your final score will be composed of homework and exam scores according to the following percentages: 10% Quizzes, 20% Homework, 20% Midterm 1, 20% Midterm 2, 30% Final Exam. What constitutes an A, B, etc. will be determined at the end once all scores have been totaled, so there is no set scale. However, I'll try to give a sense of where you stand throughout the quarter.

University Policies

Students are required to abide by Northwestern University's academic integrity policy, which can be found at http://www.northwestern.edu/provost/students/integrity/. Failure to adhere to this policy will likely result in a failing grade in the class and/or expulsion from the University.

Any student requesting accommodations related to a disability or other condition is required to register with AccessibleNU (847-467-5530) and provide professors with an accommodation notification from AccessibleNU, preferably within the first two weeks of class. All information will remain confidential.