MATH 110 - PRACTICE FINAL
LECTURE 1, SUMMER 2009
August 6, 2009

As on the practice midterm, the final will consist of much fewer problems than the number given here — probably six. These problems range from easy to medium to "almost"-hard, but I don't think any are as difficult as the starred problems from the practice midterm.

All vector spaces can be assumed to be nonzero and finite-dimensional over the field of real numbers or the field of complex numbers. Have fun!

1. Axler, 6.18 (Hint: By 6.17 it is enough to show that every vector in null P is orthogonal to every vector in range P — use 6.2 to show this)

2. Axler, 6.20

3. Suppose that V is an inner-product space and that U and W are subspaces of V. Show that $U \subseteq W$ if and only if $W^{\perp} \subseteq U^{\perp}$.

4. Suppose that S and T are self-adjoint operators on an inner-product space V. Show that ST is self-adjoint if and only if S and T commute.

5. Axler, 7.6

6. Suppose that V is an inner-product space. Show that the only positive isometry on V is the identity operator. (Hint: Spectral Theorem)

7. Axler, 7.3

8. Suppose that T is an invertible normal operator on a 3-dimensional complex vector space V. Show that T has at least 6 square roots. (Hint: Any nonzero complex number has 2 square roots)

9. Axler, 7.8

10. Axler, 8.5

11. Let V be a vector space and let T be an operator on V. Suppose that λ and μ are distinct eigenvalues of T. Show that $T - \lambda I$ is invertible when restricted to the generalized eigenspace corresponding to μ. (Hint: Use the fact that if N is nilpotent, then $N + I$ is invertible)

12. Suppose V is a 5-dimensional complex vector space and T is an operator on V such that $T^3 = 0$ and $\dim \operatorname{range} T = 3$. Find the characteristic polynomial, minimal polynomial, and Jordan form of T.

13. Show that for any operator T on a complex vector space for which $-1/4$ is not an eigenvalue, there exists an operator S such that $S^2 + S = T$. (Hint: For any nilpotent operator N and scalar $a \neq -1/4$, there exists an operator R such that $R^2 + R = N + aI$)

14. Suppose V is a 6-dimensional complex vector space. Write down all the possible Jordan forms of an operator which has characteristic polynomial $(z + 2)^4(z - 1)^2$ and minimal polynomial $(z + 2)^2(z - 1)$.