

Northwestern University

MATH 320-1 Midterm 2
Fall Quarter 2023
November 14, 2023

Last name: _____ Email address: _____

First name: _____ NetID: _____

This is just the cover page Crowdmark forces me to put on the front. The only thing necessary to fill out is your name. No email address nor NetID is actually needed.

There are five problems. Be clear in your work about what is scratch work and what is what you actually want graded. There are three extra pages towards the back; if you use these pages for work you want graded, be sure to indicate to which problem the work corresponds.

2. (10 points) Using the ϵ - δ definition, show that

$$\lim_{x \rightarrow 2} (\sqrt{x} + x^2 + \pi) = \sqrt{2} + 4 + \pi.$$

(Consider the function of which we are taking the limit here as having domain $(0, \infty)$ in order to ensure that \sqrt{x} actually exists.) You'll likely need $|\sqrt{a} - \sqrt{b}| \leq \sqrt{|a - b|}$ at some point.

3. (10 points) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and that

$$A = \{x \in \mathbb{R} \mid f(x) < x\}$$

is bounded and nonempty, so that $c = \sup A$ exists. Show that if $f(c) > c$, then there exists $\delta > 0$ such that $f(x) > x$ for $x \in (c - \delta, c + \delta)$, and use this to conclude that we must in fact have $f(c) \leq c$.

4. (10 points) Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x\sqrt{x+1} & \text{if } x \geq 0 \\ x^2 + x & \text{if } x \text{ is rational and negative} \\ -x^2 + x & \text{if } x \text{ is irrational and negative} \end{cases} .$$

Show that f is differentiable at 0 and not continuous at -1 .

5. (10 points) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable and that $f(x)f'(x) = 0$ for all $x \in \mathbb{R}$. Show that f is a constant function. Hint: For any $x, a \in \mathbb{R}$, consider $f(x)^2 - f(a)^2$.

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