PROBLEM SET 1

Problem 1: Let $\mathcal{A}$ be a finite set and recall the metric we defined on $\mathcal{A}^{\mathbb{Z}}$:

$$d(x,y) = \begin{cases} 2^{-k} & k \geq 0 \text{ maximal for which } x_i = y_i \text{ for all } |i| < k \\ 0 & \text{if } x = y \end{cases}.$$

(1) Show that d is a metric.

(2) Show that the topology induced by the above metric coincides with the product topology coming from using the discrete topology on $\mathcal{A}$.

(3) Show that $\mathcal{A}^{\mathbb{Z}}$ is compact (without using Tychonoff!).

Problem 2: Show that for $\mathcal{A}$ finite with $|\mathcal{A}| \geq 2$, the space $\mathcal{A}^{\mathbb{Z}}$ is perfect.

Problem 3: Find an example of a compact subset $X \subset \mathcal{A}^{\mathbb{Z}}$ such that $\sigma(X) \subset X$ but $\sigma(X) \neq X$.

Problem 4: Show that for any subshift X, cylinder sets generate the topology on X.

Problem 5: Let X_{11} be the subshift in $\{0,1\}^{\mathbb{Z}}$ defined by forbidding the word 11. What is the sequence $|\mathcal{L}_k(X_{11})|$?

Problem 6: Show that there are uncountably many distinct subshifts in $\{0,1\}^{\mathbb{Z}}$.

Problem 7: Build a nonempty subshift X which has no periodic points.

Problem 8: Show that if (X, σ_X) and (Y, σ_Y) are subshifts, then $(X \times Y, \sigma_X \times \sigma_Y$ can be identified with (i.e. is topologically conjugate to) a subshift. Show also that (X, σ_X^n) is topologically conjugate to a subshift for any $n \geq 1$.