Perverse sheaves, contact homology and cubical approximations

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Knot contact homology is an interesting geometric invariant of knots K in $\mathbb{R}^3$ defined by Floer-theoretic counting of pseudoholomorphic disks in the sphere conormal bundle of K in $T^*\mathbb{R}^3$. In its simplest form, this invariant was introduced by L. Ng and has been extensively studied in recent years by means of symplectic geometry and topology. In this talk, we will give a new, purely algebraic construction of knot contact homology based on the homotopy theory of (small) dg categories. For a link L in $\mathbb{R}^3$, we define a dg k-category $\mathscr{A}_L$ with a distinguished object, whose quasi-equivalence class is a topological invariant of L. In the case when L is a knot, the endomorphism algebra of the distinguished object of $\mathscr{A}_L$ coincides with a geometric dg algebra model of the knot contact homology of L constructed by Ekholm, Etnyre, Ng and Sullivan (2013). The input of our construction is a natural action of the Artin braid group B_n on the category of perverse sheaves on a two-dimensional disk with singularities at n marked points, studied by Gelfand, MacPherson and Vilonen (1996). Time permitting, we will also discuss a possible generalization of contact homology theory to arbitrary spaces in terms of homotopy theory of cubical simplicial sets.