445-2 Differential Geometry
Northwestern University
Homework 2
Due January 29 in class.

1) Let $X_{p,q}$ be the complex manifold diffeomorphic to $S^{2p-1} \times S^{2q-1}$ which you constructed in Homework 1, problem 1. For which values of p, q is $X_{p,q}$ Kähler?

2) Let $Y = X/\sigma$ be the complex manifold constructed in Homework 1, problem 3.

(a) Show that Y is Kähler.

(b) Calculate the Hodge numbers $h^{1,0}(Y), h^{0,1}(Y), h^{2,0}(Y), h^{1,1}(Y)$ and $h^{0,2}(Y)$.

3) Let X be a compact complex manifold. Prove that the map that associates to a smooth real 1-form its $(0,1)$ part induces an injective map

$$H^1(X,\mathbb{R}) \hookrightarrow H^{0,1}_{\bar{\partial}}(X),$$

from deRham cohomology to Dolbeault cohomology. Show that this implies that $b_1(X) \leqslant 2h^{0,1}(X)$.