**445-2 Differential Geometry**
**Northwestern University**
**Homework 5**
**Due February 19 in class.**

1) Let $(X, \omega)$ be a Kähler manifold of complex dimension $n \geqslant 2$. Prove that it is Kähler-Einstein if and only if

$$\mathrm{Ric}(\omega) = \frac{R}{n}\omega,$$

where $R = g^{i\bar{j}} R_{i\bar{j}}$ is the scalar curvature.

2) Let $(X, \omega)$ be a compact Kähler manifold of complex dimension $n$. Assume that $\omega$ has constant scalar curvature, and that $c_1(X) = \lambda[\omega]$ for some $\lambda \in \mathbb{R}$ (possibly zero).

(a) Prove that $\omega$ must be Kähler-Einstein.

(b) Find a simple example of a compact Kähler manifold $(X, \omega)$ with constant scalar curvature which is not Kähler-Einstein.

3) Let $X$ be a compact Kähler manifold with positive first Chern class $c_1(X) > 0$. Using the Calabi-Yau theorem, prove that $H^{1,0}_{\bar{\partial}}(X) = 0$.