1) Poisson bracket \( \xi, \eta \)

on \( \mathcal{Q} \)

Examples: \( T^*Y, \mathbb{C}^{2n}, \text{Sp} G V \) 

\( X = \mathbb{V} / G \)

\( g^* \) coadjoint orbits

\( \mathbb{E} \) Poisson variety, \( \mathbb{E} \) Poisson subvar \( Y \)

\( \mathbb{E} Y \) is a Poisson ideal: \( \mathfrak{I}_Y, \mathfrak{g}^* \subseteq \mathfrak{I}_Y \) 

(Poisson scheme...)

Particular case: \( g \) semisimple, \( \text{Nil}(g) \subseteq g \)

Killing: \( \mathfrak{g} = g^* \), \( \text{Nil}(g) = \overline{G \cdot e} \)

\( e \) "principal nilpotent"

\( \text{Nil}(g) \) singular.

Symplectic resolutions: \( \mathbb{X} \rightarrow X \)

\( \mathbb{E} \) proper, Poisson

\( X \) symplectic

\( \mathbb{E} \mathcal{T}^*(G/B) \rightarrow \text{Nil}(g) \)

\( B \subseteq G \) Borel

\( \mathbb{C}^{2n} / T \), \( T \subseteq \text{Sp}(2n) \)

Particular case: \( \mathbb{C}^2 \)
cyclic $\langle (5 \ 0) 
\downarrow
(0 \ 5^{-1}) \rangle$

dihedral $\langle (5 \ 0) 
\downarrow
(0 \ 5^{-1}) \rangle$

$S = e^{2\pi i/n}$

$SL(2, C) \supset SU(2, C) \rightarrow SO(3, R)$

rotation groups
of Platonic solids
(3 of them):
tetrah., cube $\leftrightarrow$ octah.,
dodec. $\leftrightarrow$ icos.

Poisson traces $Q \rightarrow C$

$\phi (\xi, f, g, S) = 0$

$HP_0 (Q_x) = Q_x / \Xi \Omega_x, \Omega_x \notin \{P. \text{ traces}\}$

$HP_0 (Q_x)^* = X$-affine Poisson

affine symplectic case:

$HP_0 (Q_x) \sim \dim H_{DR} (X)$

$f \mapsto f \text{ vol } x \in \sum^{\dim}_x (X)$

$HP_0 (Q_{g^*}) = (\text{Sym } g_y)_y = (\text{Sym } g_y) = \Theta g^* / G$
So $G.e \leq \mathfrak{g}^*$ : $e \in \mathfrak{g}^*$

$H^0_\mathcal{P} \left( \overline{G.e} \right) = \overline{G.e} \cong \mathbb{C}

\textbf{Comment} \ x \ \text{Poisson, } \mathbb{C}^* \text{ contracting action}
\text{ i.e. } \mathfrak{g}_x \text{ would graded}
\ (\text{as a } \text{Poisson alg})

$\mathfrak{g}_x \rightarrow (\mathfrak{g}_x)_0 \ (= \mathbb{C})$
proj

If $x, y \in \mathfrak{x} \ 0$-dim leaf,
\sympl
\alpha_x: \mathfrak{x} \rightarrow \mathbb{C} \text{ is a Poisson trace}

\underline{Nontrivial} \ : \ \text{At } e \in \mathfrak{g}_y, \ \text{of f.d. s.-s.,}
\text{there is a Kostant Slodowy slice}
\ e \in \mathfrak{s}_e \subseteq \mathfrak{g}_y \text{ transverse to } \mathfrak{s}_e
g.e

$T^*G/B \xrightarrow{\phi} \text{Nil}(\mathfrak{g}) = N$
\\text{at } e
e

$p^{-1}(\mathfrak{s}_e \cap N) \rightarrow \mathfrak{s}_e \cap N$
\textbf{Symplectic Resolution}
Fact: This is a symplectic resolution.

For the appropriate Poisson structures that you can get by Hamiltonian reduction.

\[ HP_0(\mathcal{O}_{\text{SeNN}}) \cong \text{dim} \mathcal{E} \quad (\text{SeNN}) \]

\[ \cup \]

\[ H_{\text{top}}(\rho^{-1}(e)) \quad \text{Springer fiber} \]

**Conjecture**

This holds for any symplectic resolution with \( X \) affine:

\[ HP_0(\mathcal{O}_X) \cong H_{\text{top}}(X) \]

**Theorem**

If \( X \) has finitely many symplectic leaves then \( HP_0(\mathcal{O}_X) \) is f.d.

**Def**

\( X \) has fin. many sympl. leaves if

\[ X = \bigsqcup X_i, \text{ locally closed} \]

\[ X_i \subset X \text{ Poiss} \]

\( X_i \text{ symplectic} \)
Ex. \( X \rightarrow X \) sympl. res \( \Rightarrow \) fin many s. leaves

\( C^{2n}/G \quad G = Sp(2n) \)

Closures of Sympl. leaves are

\[
\left( C^{2n} \right)^K / \left( N(K)/K \right)
\]

Def. \( K \subset G \) parabolic if \( \exists v \in C^{2n} \ Stab_G(v) = K \)

Generally, \( \text{dim } v \geq 2 \), \( V \) symplectic \( \Leftrightarrow \)

\( V/G \) does not admit symplectic resolution except in special cases.

To prove theorem: rewrite \( H^0(\Omega_X) \) using diff. equs. \( f \in \Omega_X \); \( \xi_f = \delta f \), \(-\frac{1}{2}\)

Hamiltonian vector field \( \xi \in H(X) \)

\( H^0(\Omega_X)^* \cong \text{Hom} (\Omega^* \otimes X, H(X)) \)

solution space of d.e.
\[ \text{Def} \quad \mathbb{X} \text{ smooth,} \]
\[ 1 \in \mathcal{M}(x) := \mathcal{D}_x \mathcal{H}(x) \mathcal{D}_x \]
\[ \mathcal{H} \mathcal{P}_0 (\mathcal{O}_x) = \mathcal{M}(x) \otimes_{\mathcal{D}(x)} \mathcal{O}_x \]
\[ \mathcal{H} \mathcal{P}_0 (\mathcal{O}_x)^* = \mathcal{M}(x) \]
\[ = \text{Hom} (\mathcal{M}(x), \mathcal{O}_x^*) \]
\[ \mathcal{D}_x \]

Thus
\[ \pi_0 : V \rightarrow \mathfrak{p} \quad \pi_0 (\mathcal{M}) = \mathcal{M} \otimes \mathcal{O}_V \]

Thus
If \( X \) has finite dimension symplectic leaves then \( \mathcal{M}(x) \) is holonomic.