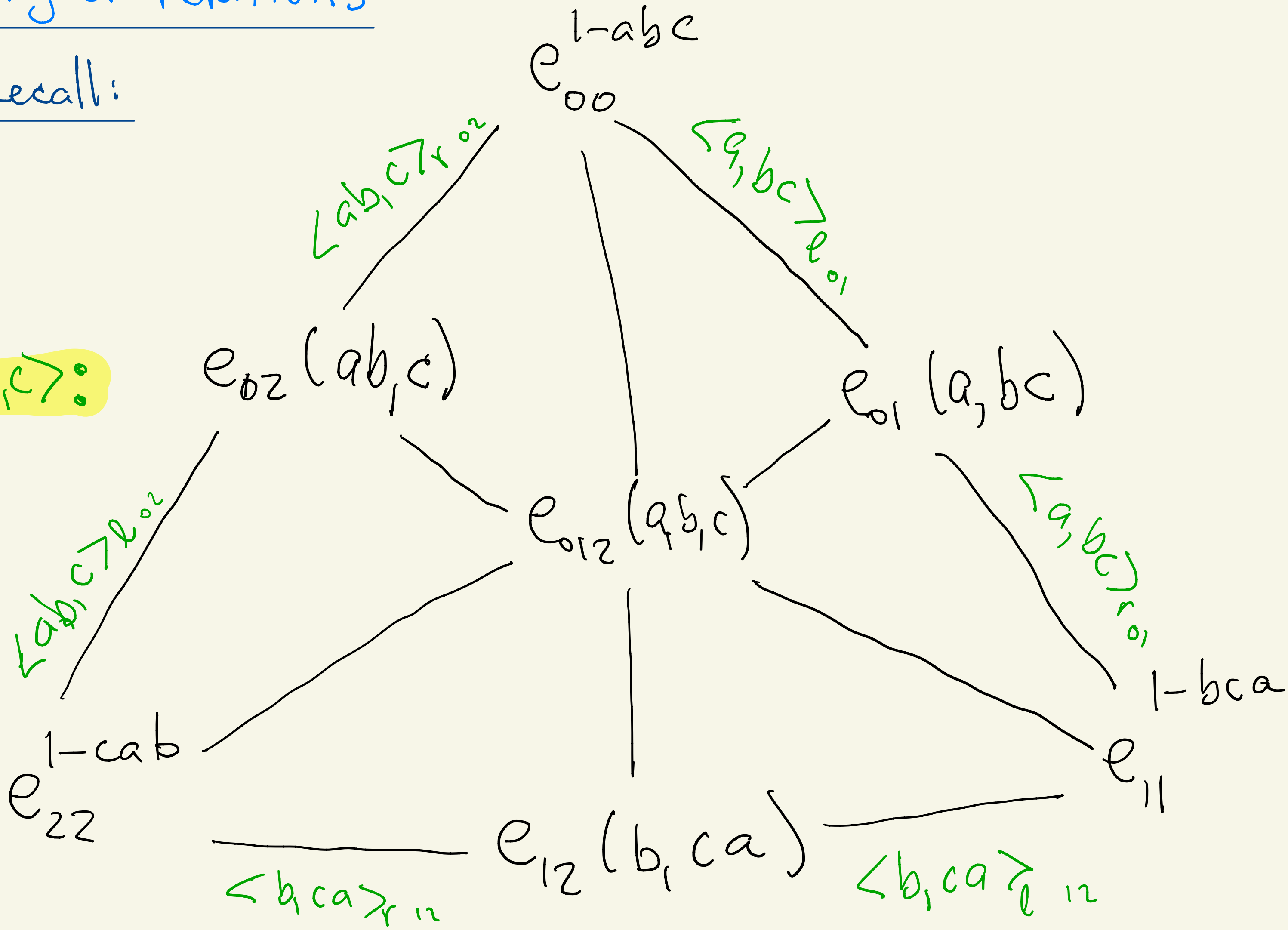
Higher relations

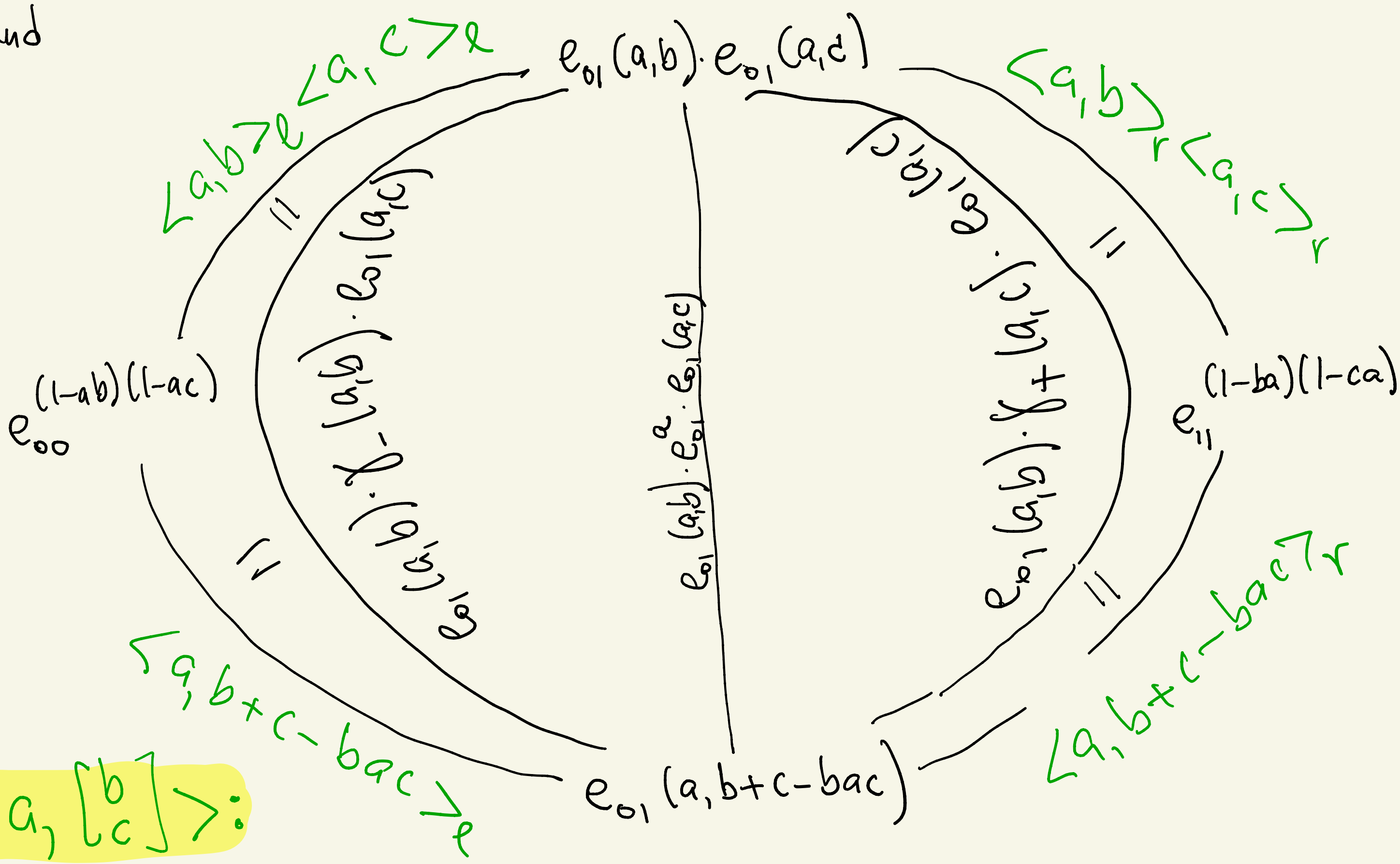
Recall:

$\langle a,b,c\rangle$:



and

$\langle a, \begin{bmatrix} b \\ c \end{bmatrix}\rangle$:

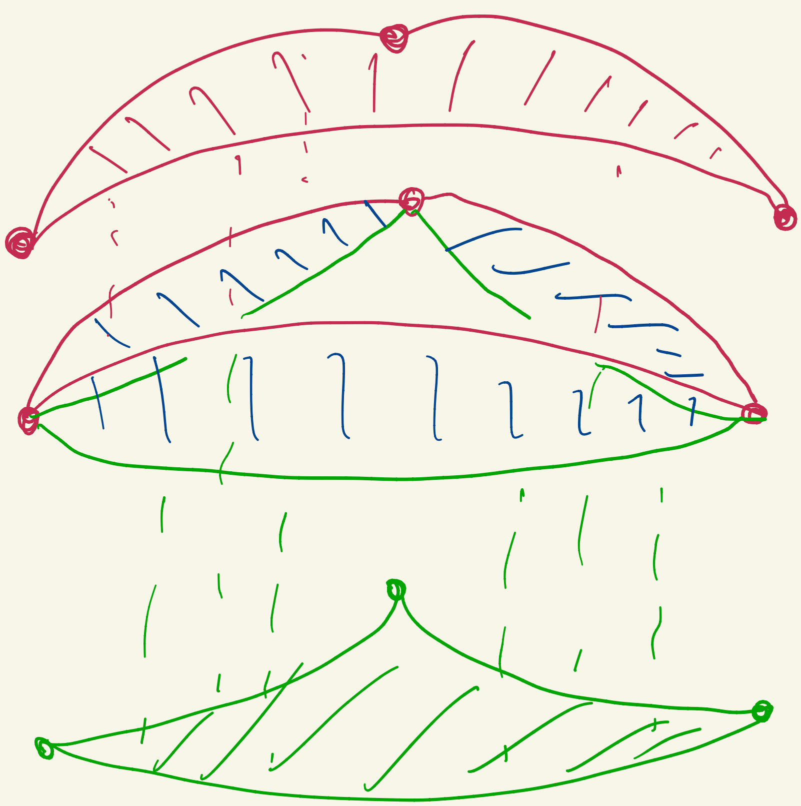


Here $\gamma_{\pm}(a,x) = \mathrm{Ad}^{-1}_{\begin{bmatrix} 1 & -a \\ -x & 1 \end{bmatrix}} X_{\pm}(a,x)$

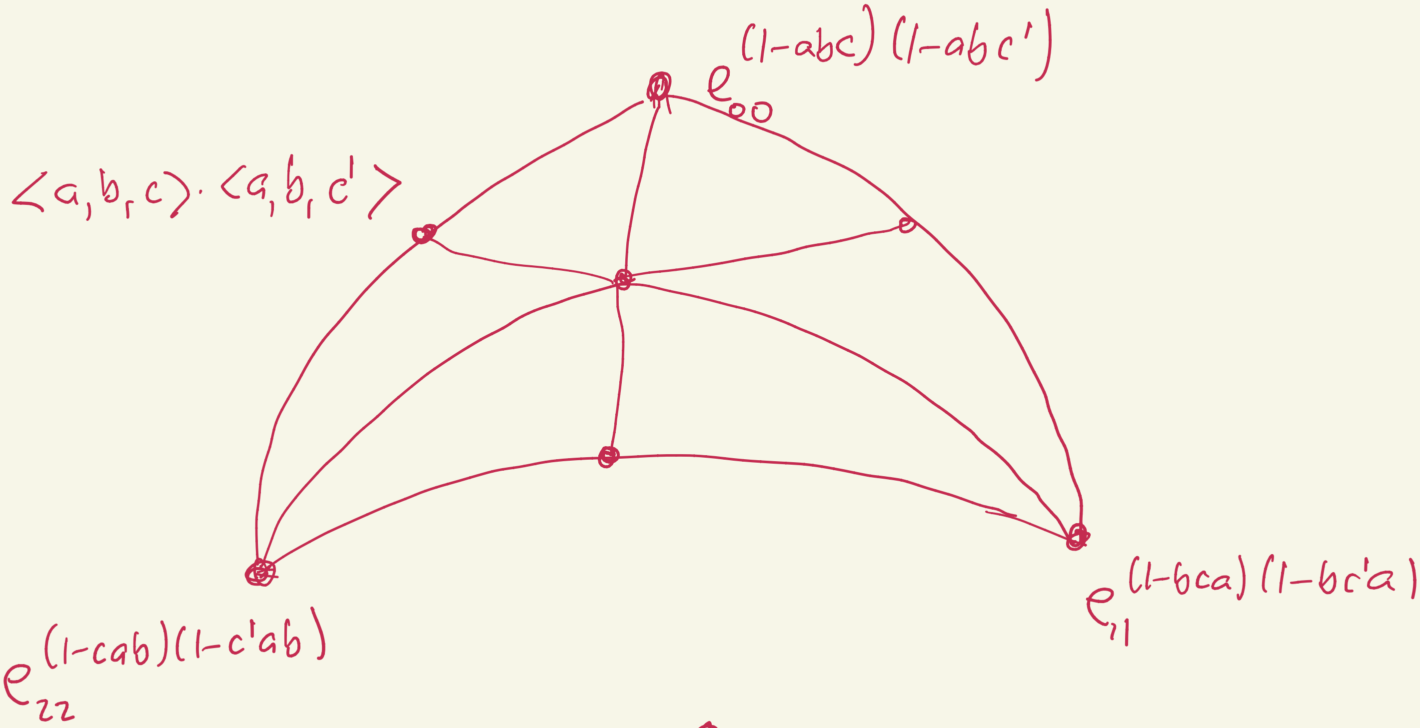
$$X_{\pm}(a,x) = \mathrm{Ad}_{\begin{bmatrix} 1 & \pm a \\ 0 & 1 \end{bmatrix}} \begin{pmatrix} 1 & -a(1-xa)^{-1} \\ 0 & 1 \end{pmatrix}$$

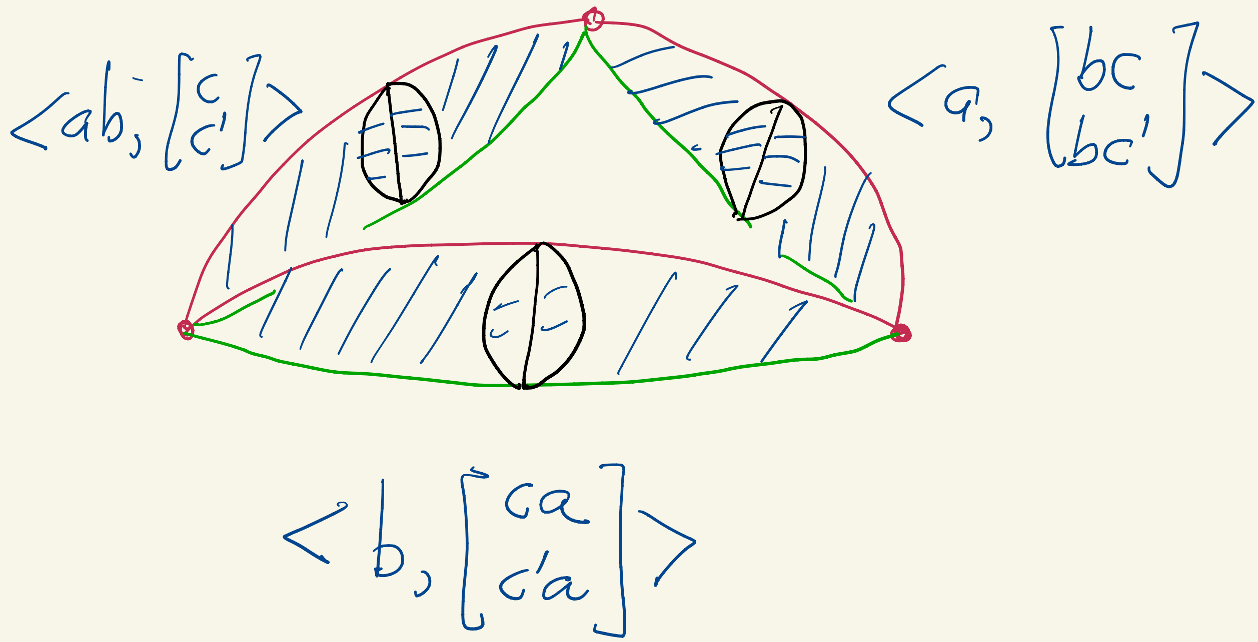
The higher homotopies should start with:

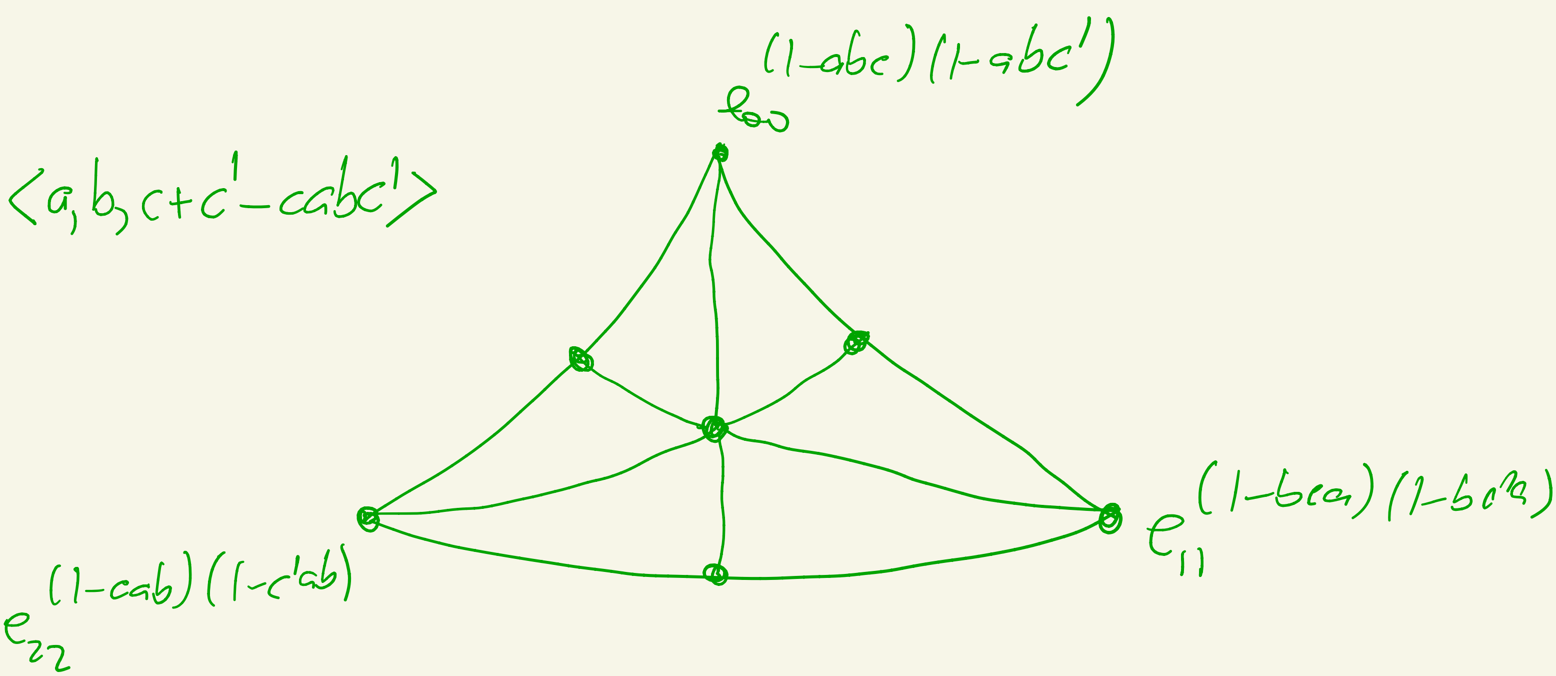
1



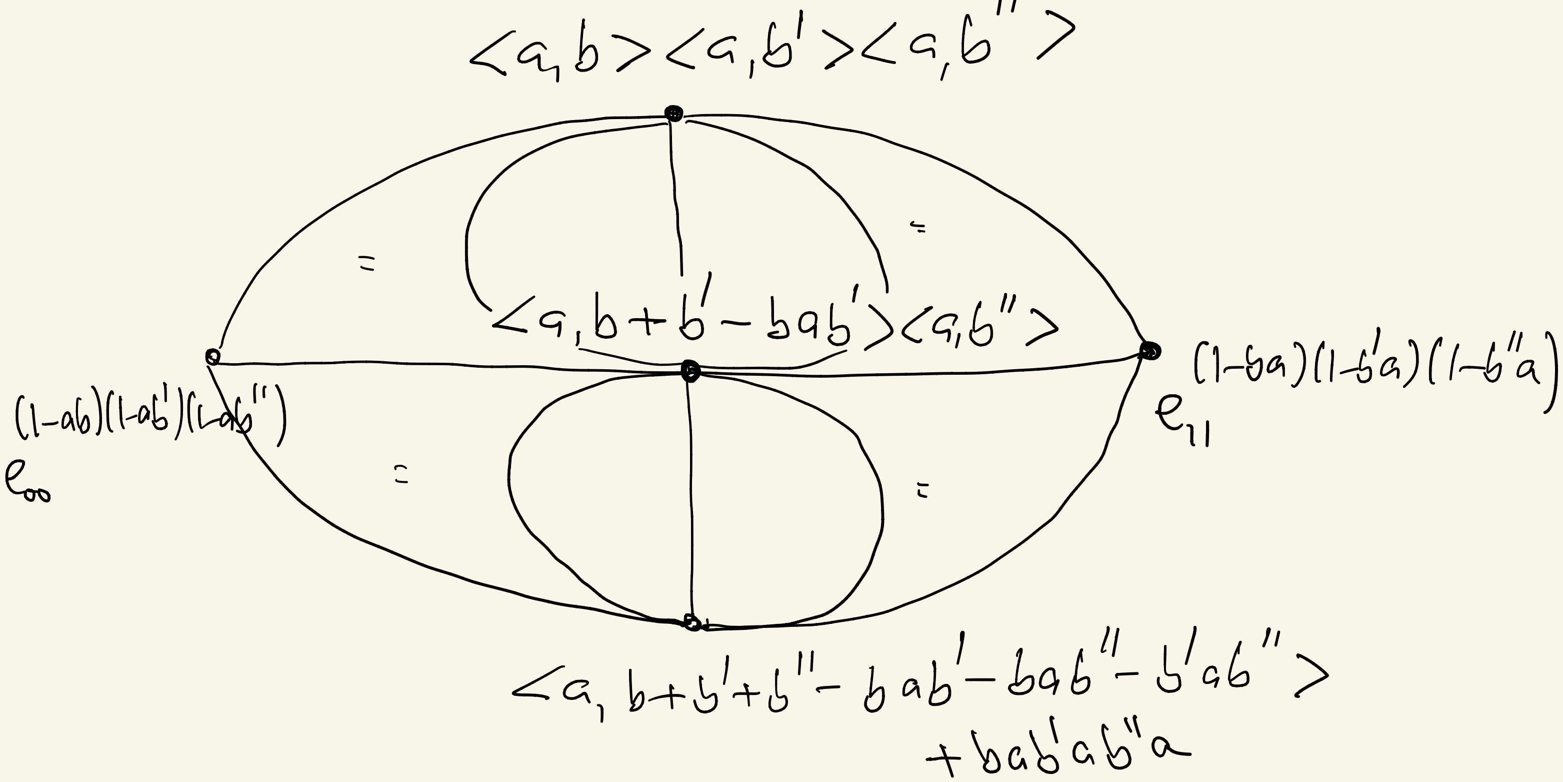
where


$e_{00}^{(1-abc)(1-abc')}$
$\langle a,b,c\rangle \cdot \langle a,b,c'\rangle$
$e_{11}^{(1-bca)(1-bc'a)}$
$e_{22}^{(1-cab)(1-c'ab)}$

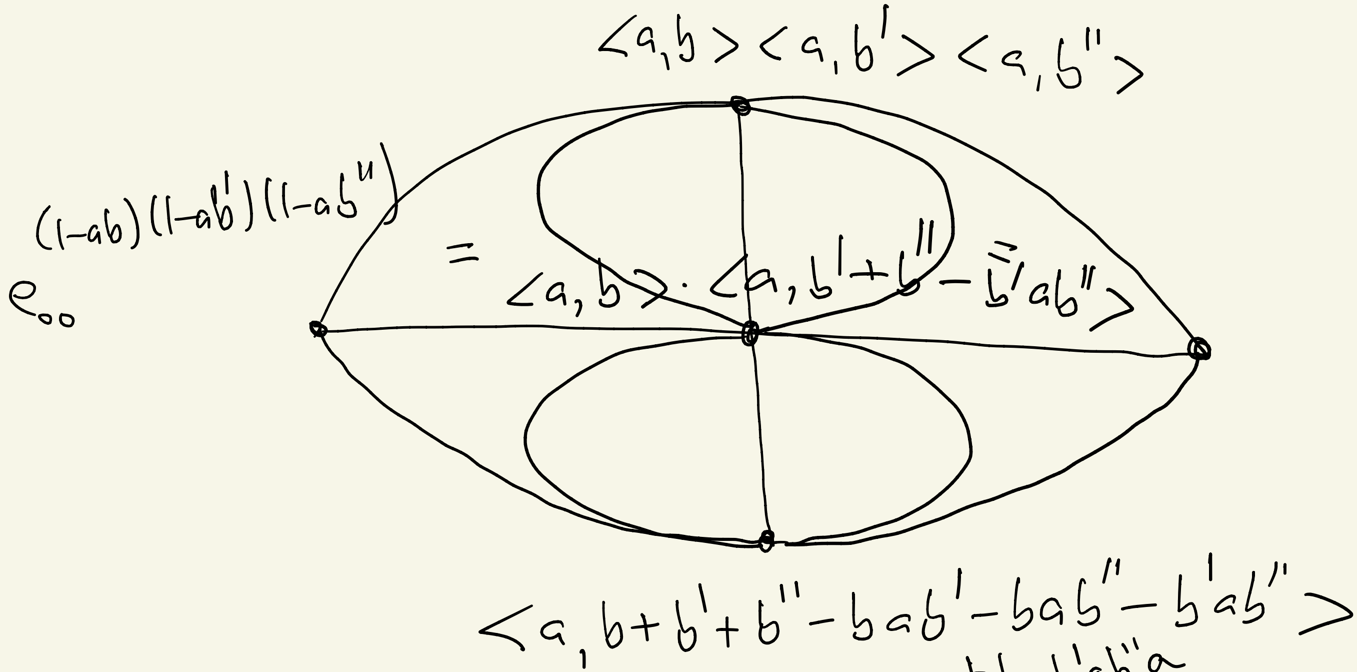

$\langle ab, \begin{bmatrix} c \\ c' \end{bmatrix} \rangle$
$\langle a, \begin{bmatrix} bc \\ bc' \end{bmatrix} \rangle$
$\langle b, \begin{bmatrix} ca \\ c'a \end{bmatrix} \rangle$


$e_{00}^{(1-abc)(1-abc')}$
$\langle a, b, c+c'-cabc' \rangle$
$e_{11}^{(1-bca)(1-bc'a)}$
$e_{22}^{(1-cab)(1-c'ab)}$

2 (Associativity of $\langle a, \begin{bmatrix} b \\ b' \end{bmatrix} \rangle$)

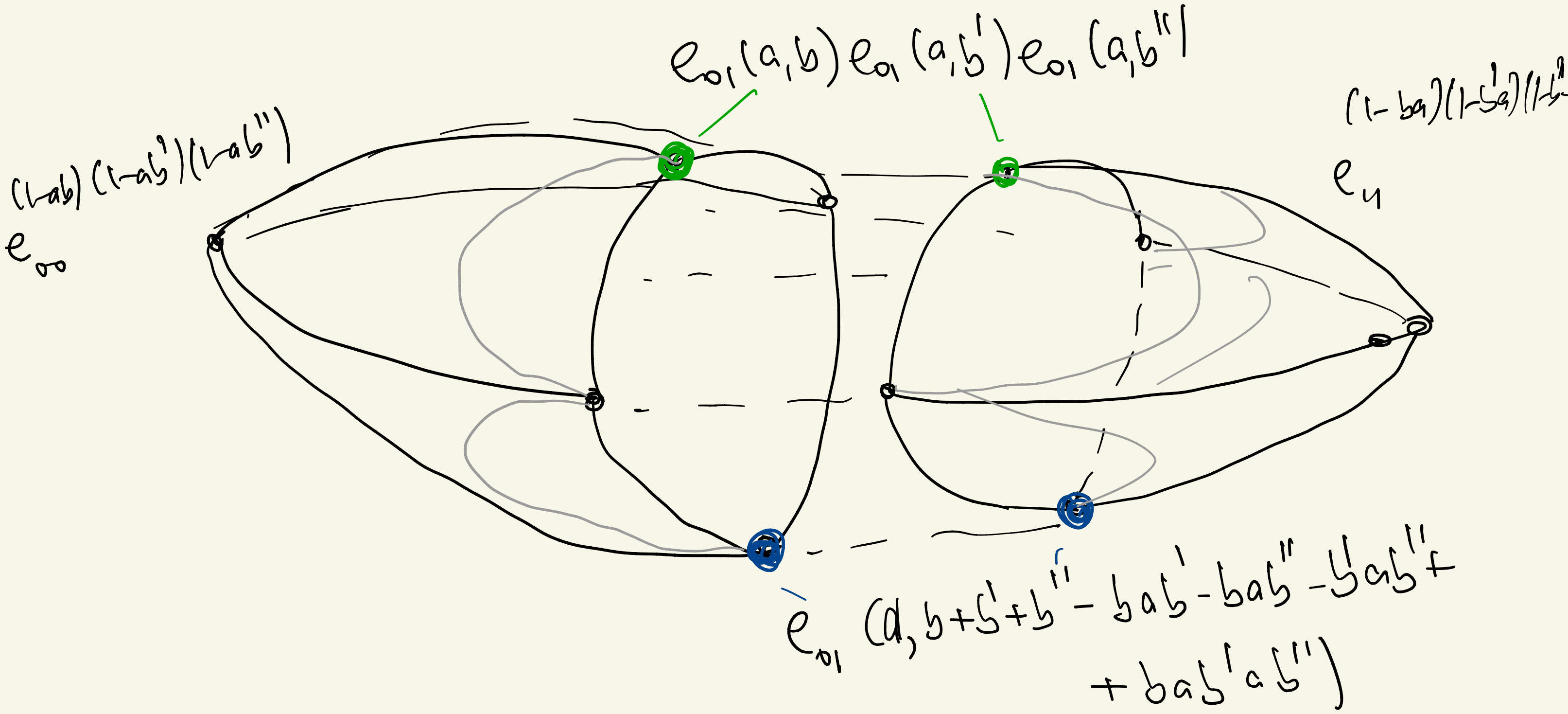


and



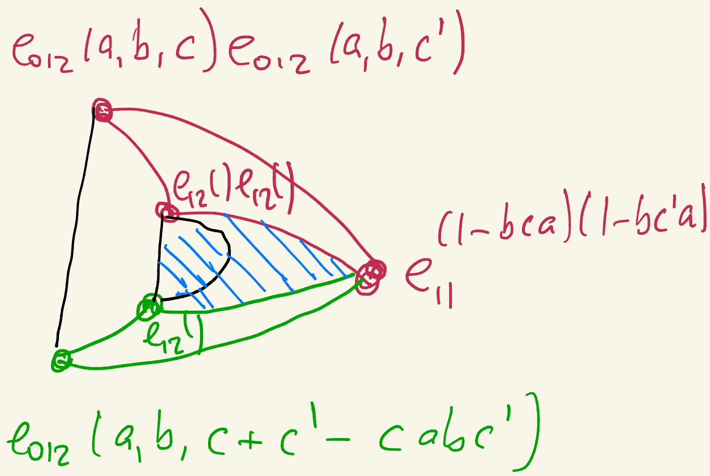
Should be homotopic.

$\langle a, \begin{bmatrix} b \\ b' \\ b'' \end{bmatrix} \rangle$.



Will probably be done section-by-section.

In case of (1):



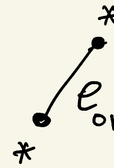
The above should be the beginning of

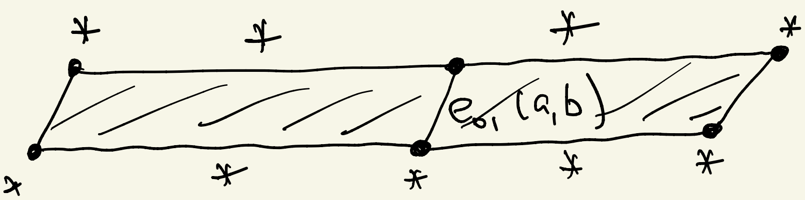
$$\operatorname*{hocolim}_{\Lambda^{op}} CL_{*,\bullet}(A) \to Sing_{*,\bullet}(|B_* GL(A\{\Delta^\bullet\})|)$$

(See: Notes on Nonlinear cyclic homology).

(We need a better handle on the (homotopy trivial) action of permutations).

<u>Rmk</u>: Above, we drew the pictures of $\langle a, b, c\rangle$, $\langle a, \begin{bmatrix} b \\ c \end{bmatrix}\rangle$, etc. in $|GL(A\{\Delta^\bullet\})|$ rather than in $|B_* GL(A\{\Delta^\bullet\})|$. The new pictures should follow; they should be somewhat different, in dimension one up. E.g. for $a \in M(A)$, $1-a \in GL$:

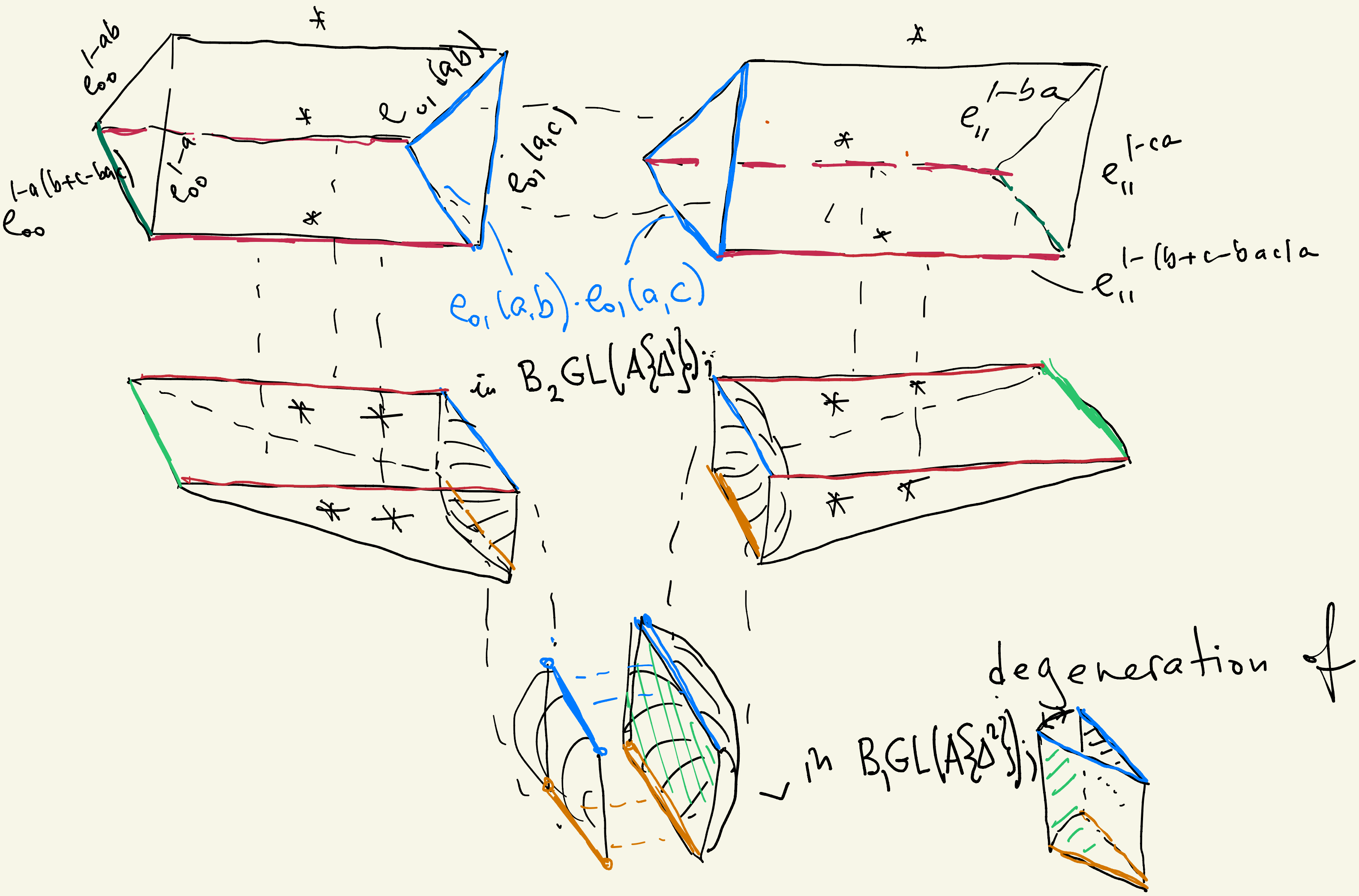
$\bullet\ e_{00}^{1-a}$ in $|GL(A\{\Delta^\bullet\})|$;  in $|B_* GL(A\{\Delta^\bullet\})|$

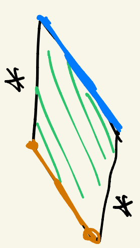
$\langle a,b\rangle$: e_{00}^{1-ab}  e_{11}^{1-ba} (* the marked pt in $C_{0,\bullet}$)

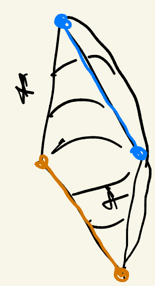
$\langle a,b\rangle$: $\Delta^1 \times \Delta^1 \longrightarrow |B_* GL(A\{\Delta^\bullet\})|$ $\quad \langle a,b\rangle \in Sing_{1,1}$

Here and everywhere: still have to take care of the discrepancy e_{00}^{1-ab} vs e_{11}^{1-ba}.

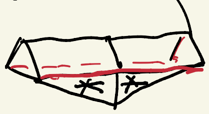
$\langle a, \begin{bmatrix} b \\ c \end{bmatrix} \rangle$ becomes:



 $e_{01}(a,b)\cdot e_{01}^{a}\cdot e_{01}(a,c)$;

 $e_{01}(a,b)\cdot \gamma_-(a,c)\cdot e_{01}(a,c)$

in $B_1GL(A\{\Delta^1\})$

Get a map  $\langle a, \begin{bmatrix} b \\ c \end{bmatrix} \rangle : \Delta^2 \times \underset{\substack{\uparrow \\ \mathrm{Sing}_{2,1}}}{\Delta^1} \longrightarrow |B_*GL(A\{\Delta^\bullet\})|$

Rmk Both $CL_{*,\cdot}$ and $B_*GL(A\{\Delta^\cdot\})$ are nerves of simplicial (cyclic) categories. (Up to a minor point ☆).

Perhaps one should replace Nerve by $\mathfrak{N}$, the coherent nerve.

Will it change the answer?

Can one construct, instead of the above,

$$\mathfrak{N}(\ldots) \longrightarrow \mathfrak{N}GL(A\{\Delta^\cdot\})?$$

(How to modify this to include $\operatorname{hocolim}_{\Lambda^{op}}$?)